Further results for the two-loop Lcc vertex in the Landau gauge

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ABSTRACT: In the previous paper hep-th/0604112 we calculated the first of the five planar two-loop diagrams for the Lcc vertex of the general non-Abelian Yang-Mills theory, the vertex which allows us in principle to obtain all other vertices via the Slavnov-Taylor identity. The integrand of this first diagram has a simple Lorentz structure. In this letter we present the result for the second diagram, whose integrand has a complicated Lorentz structure. The calculation is performed in the D-dimensional Euclidean position space. We initially perform one of the two integrations in the position space and then reduce the Lorentz structure to D-dimensional scalar single integrals. Some of the latter are then calculated by the uniqueness method, others by the Gegenbauer polynomial technique. The result is independent of the ultraviolet and the infrared scale. It is expressed in terms of the squares of spacetime intervals between points of the effective fields in the position space — it includes simple powers of these intervals, as well as logarithms and polylogarithms thereof, with some of the latter appearing within the Davydychev integral $J(1,1,1)$. Concerning the rest of diagrams, we present the result for the contributions correponding to third, fourth and fifth diagrams without giving the details of calculation. The full result for the Lcc correlator of the effective action at the planar two-loop level is written explicitly for maximally supersymmetric Yang-Mills theory.

KEYWORDS: BRST Quantization, BRST Symmetry, Extended Supersymmetry, Supersymmetric gauge theory.

Contents

1. Introduction

It has been shown in refs. [1–3] that the effective action of dressed mean fields for $\mathcal{N} = 4$ super-Yang-Mills theory does not depend on any scale, ultraviolet or infrared. These results were derived from the results of refs. [4–8]. Scale independence suggests that kernels of these dressed mean fields can be analyzed by the methods of conformal field theory. We started to investigate the simplest scalar vertex Lcc in the Landau gauge in ref. [9], where c is a real ghost field and L is the auxiliary field which couples at the tree level to the BRST [10, 11] transformation of the c field. This vertex is simple in the Landau gauge where it is totally finite for $\mathcal{N} = 4$ super-Yang-Mills theory. By solving Slavnov-Taylor (ST) identity [12–17], all other vertices in that theory can be derived from this vertex [4]. The ST identity is a consequence of the BRST symmetry of the classical action [10, 11]. Recently, by using unitarity methods it has been demonstrated up to the four-loop level that

the only off-shell conformal integrals in the momentum space contribute in the maximally-helicity-violating (MHV) four-particle amplitudes [18], and iterative structure has been conjectured for all the MHV amplitudes [19].

As has been found in ref. [20], all the contributions to the off-shell four-point correlator of gluons in that theory up to three-loop level (at least) are equivalent to ladder integrals (UD integrals) calculated in the momentum space by Usyukina and Davydychev in refs. [21, 22]. They are expressed in terms of certain functions (UD functions). It is known from refs. [20−23] that UD functions are conformally invariant in the momentum space. Fourier transform of UD functions in the momentum space can be expressed in terms of the UD functions in the position space which possess the same property of the conformal invariance but this time with the arguments of the position space. On the other hand, the scale independence of the effective action of the dressed mean fields is a consequence of the vanishing of the beta function in the maximally supersymmetric Yang-Mills theory and the vanishing in turn is a consequence of the conformal symmetry of the theory. All this suggests that correlators of the dressed mean fields can be analyzed by the technique of the conformal field theory.

To solve the ST identity and to check the conformal invariance of the effective action of the dressed mean fields explicitly, we should work in the position space. The two-loop contribution contains five diagrams. We calculated the first contribution in the previous paper [9]. Now we calculate the second contribution that corresponds to the diagram (b) of ref. [9], give the result for the rest of the diagrams, and present the full result for the planar two-loop contribution to the Lcc correlator of the effective action in the Landau gauge for $\mathcal{N} = 4$ supersymmetric Yang-Mills theory. The notation used here is the same as in ref. [9]. The Lcc vertex is superficially convergent in the Landau gauge. This fact can be checked by index counting and by noting that two derivatives from the ghost propagators can always be integrated out of the diagram due to the transversality of the gauge propagator. It means that the field c does not have renormalization in the Landau gauge. Formally, this result holds to all orders of perturbation theory due to the so called antighost equation [24]. In the nonsupersymmetric theories this vertex is not finite and a calculation of the anomalous dimension of operator cc has been performed in [25, 26]. Finiteness of this Lcc vertex at planar two loop level in Landau gauge in maximally supersymmetric Yang-Mills theory is equivalent to the cancellation of poles between fourth and fifth diagrams of ref. [9] that is, in turn is equivalent to the vanishing of the β function of the gauge coupling in that theory.

Knowing the structure of the Lcc vertex one can obtain the structure of other irreducible vertices by solving the ST identity. The algorithm can be applied also to other theories different from the theory in consideration. Among possible applications are $\mathcal{N} = 8$ supergravity, Chern-Simons theory, string field theory, massless gauge theory near fixed points in the coupling space, and topological field theories in higher dimensions.

The paper has the following structure. In section 2 we write the integral expression that corresponds to the diagram (b), and analyze it by dividing it in three parts. In section 3, the result for the first part is reduced to single D-dimensional integrals ($D = 4 - 2\epsilon$). The analogous expressions for the second and the third part are written in A. The sum of all three parts contains single D-dimensional integrals $J(\alpha_1, \alpha_2, \alpha_3)$ with the sum of indices
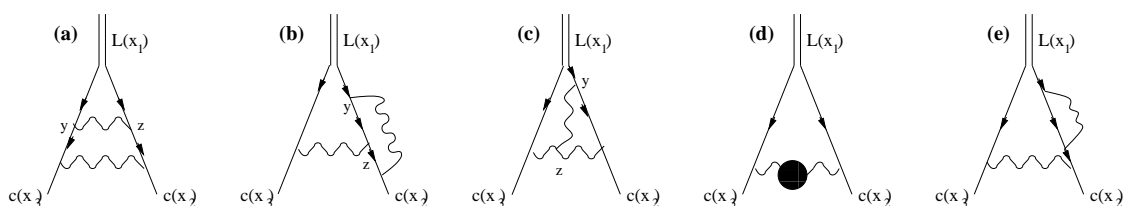
Figure 1: The two-loop diagrams for the Lcc vertex. The wavy lines represent the gluons, the straight lines the ghosts. Black disk stands for the total one-loop correction to the gluon propagator.

$\alpha_1 + \alpha_2 + \alpha_3$ in the denominators equal to $D-1$, D, $D+1$, $D+2$. In section 4 we present the part with the integrals whose sum of indices is equal to $D-1$. In B we present the part with the integrals whose sum of the indices is D, $D+1$, and $D+2$; these integrals are calculated there explicitly by using the uniqueness method and its variants [27, 28]. In section 4 we reduce the aforementioned $(D-1)$-type partial sum to explicitly known terms and to terms containing the integrals $J(1,1,1)$, $J(\epsilon, 2-3\epsilon, 1)$ and derivatives thereof — by applying formulas obtained in C where the method of integration by parts procedure (IBP) [29, 30] was used. In section 5 we obtain the entire result for the diagram (b) in D dimensions, in known terms and terms with the integrals $J(1,1,1)$, $J(\epsilon, 2-3\epsilon, 1)$ and derivatives thereof. The terms with $J(\epsilon, 2-3\epsilon, 1)$ are calculated explicitly in section 5, while derivatives of $J(\epsilon, 2-3\epsilon, 1)$ are calculated in D – in both cases those formulas of C were employed which were obtained by the Gegenbauer polynomial technique (GPT) introduced originally in refs. [31 – 34] and further developed in ref. [35]. In E, all the contributions for the diagram (b) in D dimensions are collected, and the $\epsilon \to 0$ limit is performed. In section 6 we present this result in a more explicit and shorter form. In section 7 we write the result of calculation of the rest of diagram and present the total result for the planar two-loop Lcc correlator of the effective action in the Landau gauge. In Conclusions, we comment on the result obtained and on further possible developments of the analysis performed in this paper.

2. Diagram (b)

The two-loop correction to the Lcc vertex can be represented as a sum of five diagrams depicted in figure 1. We have introduced for brevity the notation

$$[yz] = (y-z)^2, \quad [y1] = (y-x_1)^2, \dots.$$

and so on. The structure of Lorentz indices in diagram (b) is more complicated than the structure of the indices in the diagram (a) considered in ref. [9]. The algebra can be inferred immediately from the diagram (b).

$$\frac{(2z)_\sigma}{[2z]^{2-\epsilon}} \left(\frac{g_{\sigma\lambda}}{[2y]^{1-\epsilon}} + 2(1-\epsilon) \frac{(2y)_\sigma (2y)_\lambda}{[2y]^{2-\epsilon}} \right) \frac{(y1)_\lambda}{[y1]^{2-\epsilon}} \times$$

$$\times \frac{(zy)_\mu}{[zy]^{2-\epsilon}} \left(\frac{g_{\mu\nu}}{[3z]^{1-\epsilon}} + 2(1-\epsilon) \frac{(3z)_\mu (3z)_\nu}{[3z]^{2-\epsilon}} \right) \frac{(31)_\nu}{[31]^{2-\epsilon}}$$

We work in the position space, in $D = 4 - 2\epsilon$ dimensions, using the technique of dimensional regularization [36], thus maintaining the intermediate results finite. We have to calculate the $2D$-dimensional integral[1]

$$\int Dy\ Dz\ \frac{(2z)_\sigma}{[2z]^{2-\epsilon}}\left(\frac{g_{\sigma\lambda}}{[2y]^{1-\epsilon}} + 2(1-\epsilon)\frac{(2y)_\sigma(2y)_\lambda}{[2y]^{2-\epsilon}}\right)\frac{(y1)_\lambda}{[y1]^{2-\epsilon}}\times$$
$$\times\frac{(zy)_\mu}{[zy]^{2-\epsilon}}\left(\frac{g_{\mu\nu}}{[3z]^{1-\epsilon}} + 2(1-\epsilon)\frac{(3z)_\mu(3z)_\nu}{[3z]^{2-\epsilon}}\right)\frac{(31)_\nu}{[31]^{2-\epsilon}}\ .\quad(2.1)$$

By using the representation $(y1)_\lambda = (y2)_\lambda + (21)_\lambda$, after simple algebra we can represent this integral as a sum of three integrals

$$-(3-2\epsilon)\int Dy\ Dz\ \frac{(2z)_\sigma(2y)_\sigma}{[y1]^{2-\epsilon}[2z]^{2-\epsilon}[2y]^{1-\epsilon}}\frac{(zy)_\mu}{[zy]^{2-\epsilon}}\times$$
$$\times\left(\frac{g_{\mu\nu}}{[3z]^{1-\epsilon}} + 2(1-\epsilon)\frac{(3z)_\mu(3z)_\nu}{[3z]^{2-\epsilon}}\right)\frac{(31)_\nu}{[31]^{2-\epsilon}}$$
$$+(2-2\epsilon)\int Dy\ Dz\ \frac{[(2z)_\sigma(2y)_\sigma][(2y_\lambda)(21)_\lambda]}{[y1]^{2-\epsilon}[2z]^{2-\epsilon}[2y]^{1-\epsilon}}\frac{(zy)_\mu}{[zy]^{2-\epsilon}}\times$$
$$\times\left(\frac{g_{\mu\nu}}{[3z]^{1-\epsilon}} + 2(1-\epsilon)\frac{(3z)_\mu(3z)_\nu}{[3z]^{2-\epsilon}}\right)\frac{(31)_\nu}{[31]^{2-\epsilon}}$$
$$+\int Dy\ Dz\ \frac{(2z)_\sigma(21)_\sigma}{[y1]^{2-\epsilon}[2z]^{2-\epsilon}[2y]^{1-\epsilon}}\frac{(zy)_\mu}{[zy]^{2-\epsilon}}\left(\frac{g_{\mu\nu}}{[3z]^{1-\epsilon}} + 2(1-\epsilon)\frac{(3z)_\mu(3z)_\nu}{[3z]^{2-\epsilon}}\right)\frac{(31)_\nu}{[31]^{2-\epsilon}}.$$

As in the previous paper [9], we modify the ϵ's appearing in the propagators ($\epsilon \mapsto \kappa_j\epsilon$) in order to make the uniqueness method applicable and thus to make the calculations easier. Such changes do not affect our result in the $\epsilon \to 0$ limit since the integral is finite. We do the following changes: in the powers of the denominators of the four ghost propagators in eq. (2.1), we change $\epsilon \mapsto 2\epsilon$ in the first one, and $\epsilon \mapsto 0$ in the other three; in the powers of the denominators in the two gauge propagators, we change $\epsilon \mapsto 2\epsilon$ in the first and $\epsilon \mapsto 0$ in the second propagator; in the nominator of the second gauge propagator, we change $\epsilon \mapsto \bar\epsilon$, where $\bar\epsilon$ is as in the previous paper; in the nominator of the first gauge propagator, we change $\epsilon \mapsto \bar\epsilon_2$ where $\bar\epsilon_2$ is different because it is related to the requirement of keeping the transversality in the modified propagators [9]

$$\bar\epsilon = -\frac{2\epsilon}{(1-2\epsilon)}, \qquad \bar\epsilon_2 = \frac{4\epsilon}{(1+2\epsilon)}\ .\quad(2.2)$$

The above integral for the diagram (b) now acquires the following form:

$$I_b = \int Dy\ Dz\ \frac{(2z)_\sigma}{[2z]^{2-2\epsilon}}\left(\frac{g_{\sigma\lambda}}{[2y]^{1-2\epsilon}} + 2(1-\bar\epsilon_2)\frac{(2y)_\sigma(2y)_\lambda}{[2y]^{2-2\epsilon}}\right)\frac{(y1)_\lambda}{[y1]^2}$$
$$\times\frac{(zy)_\mu}{[zy]^2}\left(\frac{g_{\mu\nu}}{[3z]} + 2(1-\bar\epsilon)\frac{(3z)_\mu(3z)_\nu}{[3z]^2}\right)\frac{(31)_\nu}{[31]^2}$$
$$= -(3-2\bar\epsilon_2)I_{b,1} + (2-2\bar\epsilon_2)I_{b,2} + I_{b,3}\,,\quad(2.3)$$

---
[1]We use the D-dimensional measure $Dx \equiv \pi^{-\frac{D}{2}}d^Dx$ [9].

where the three integrals $I_{b,j}$ are

$$I_{b,1} = \int Dy\ Dz\ \frac{(2z)_\sigma (2y)_\sigma}{[y1]^2 [2z]^{2-2\epsilon} [2y]^{1-2\epsilon}} \frac{(zy)_\mu}{[zy]^2} \left(\frac{g_{\mu\nu}}{[3z]} + 2(1-\bar\epsilon)\frac{(3z)_\mu (3z)_\nu}{[3z]^2} \right) \frac{(31)_\nu}{[31]^2}\ , \quad (2.4)$$

$$I_{b,2} = \int Dy\ Dz\ \frac{[(2z)_\sigma (2y)_\sigma][(2y_\lambda)(21)_\lambda]}{[y1]^2 [2z]^{2-2\epsilon} [2y]^{2-2\epsilon}} \frac{(zy)_\mu}{[zy]^2} \left(\frac{g_{\mu\nu}}{[3z]} + 2(1-\bar\epsilon)\frac{(3z)_\mu (3z)_\nu}{[3z]^2} \right) \frac{(31)_\nu}{[31]^2} \quad (2.5)$$

$$I_{b,3} = \int Dy\ Dz\ \frac{(2z)_\sigma (21)_\sigma}{[y1]^2 [2z]^{2-2\epsilon} [2y]^{1-2\epsilon}} \frac{(zy)_\mu}{[zy]^2} \left(\frac{g_{\mu\nu}}{[3z]} + 2(1-\bar\epsilon)\frac{(3z)_\mu (3z)_\nu}{[3z]^2} \right) \frac{(31)_\nu}{[31]^2}\ . \quad (2.6)$$

In these $2D$-dimensional integrals, we first perform the Lorentz algebra only for certain subproducts in the numerators, then perform one of the two D-dimensional integrations by using the uniqueness method or its variants,[2] and then finish the Lorentz algebra. We found this method simpler than making the entire Lorentz algebra initially and then integrating the contributions. After integrating, the number of terms in the algebra is significantly reduced.

3. Reduction to single scalar integrals

Following the afore-described procedure, we can scalarize the expression for each of the three $2D$-dimensional integrals in eq. (2.3), reducing them to linear combinations of integrals $J(\alpha_1, \alpha_2, \alpha_3)$ where

$$J(\alpha_1, \alpha_2, \alpha_3) = \int Dx \frac{1}{[x1]^{\alpha_1} [x2]^{\alpha_2} [x3]^{\alpha_3}}\ .$$

For the first integral we obtain

$$\begin{aligned}
I_{b,1} = \frac{A(1, 1-2\epsilon, 1)}{4\epsilon(1-2\epsilon)^2} \Bigg\{ &\left(-\frac{6(1-\epsilon^2)(1-2\epsilon)}{[31]^2[12]^{-\epsilon}} - \frac{2(1-\epsilon^2)(1-2\epsilon)}{[31][12]^{1-\epsilon}} \right) J(1+\epsilon, 2-3\epsilon, 1) \\
&+ \frac{2(1-\epsilon^2)(1-2\epsilon)}{[31]^2[12]^{1-\epsilon}} J(\epsilon, 2-3\epsilon, 1) + \frac{2(1-\epsilon^2)(1-2\epsilon)}{[31][12]^{1-\epsilon}} J(2+\epsilon, 1-3\epsilon, 1) \\
&+ \frac{2(1-\epsilon^2)(1-2\epsilon)}{[31]^2[12]^{1-\epsilon}} J(2+\epsilon, 1-3\epsilon, 0) - \frac{(1-2\epsilon^2)(1-2\epsilon)}{[31]^2[12]^{1-\epsilon}} J(1+\epsilon, 1-3\epsilon, 1) \\
&+ \frac{6(1-\epsilon^2)(1-2\epsilon)}{[31][12]^{-\epsilon}} J(2+\epsilon, 2-3\epsilon, 1) - \frac{2(1-\epsilon^2)(1-2\epsilon)}{[31]^2[12]^{1-\epsilon}} J(1+\epsilon, 2-3\epsilon, 0) \\
&+ \frac{\epsilon - \epsilon^2}{[31][23]^{1-\epsilon}} J(2, 1-3\epsilon, 1+\epsilon) + \frac{\epsilon - \epsilon^2}{[31]^2[23]^{1-\epsilon}} J(1, 2-3\epsilon, \epsilon) \\
&- \frac{4 - 8\epsilon + 3\epsilon^2}{[31][23]^{-\epsilon}} J(2, 2-3\epsilon, 1+\epsilon) + \frac{4 - 8\epsilon + 3\epsilon^2}{[31]^2[23]^{-\epsilon}} J(1, 2-3\epsilon, 1+\epsilon) \\
&+ \left(\frac{\epsilon(1-2\epsilon)}{[31][23]^{1-\epsilon}} - \frac{4 - 10\epsilon + 6\epsilon^2}{[31]^2[23]^{-\epsilon}} - \frac{[12]\epsilon(2-3\epsilon)}{[31]^2[23]^{1-\epsilon}} \right) J(2, 2-3\epsilon, \epsilon) \\
&+ \left[-\frac{(1-2\epsilon)(1-\epsilon)}{[31]^2[23]^{1-\epsilon}} - \frac{(1-\epsilon)^2}{[31][23]^{2-\epsilon}} + \frac{[12](1-\epsilon)^2}{[31]^2[23]^{2-\epsilon}} \right] J(2, 1-3\epsilon, \epsilon)
\end{aligned}$$

---

[2] By the "variants of the uniqueness method" we mean the cases when the sum of the powers in the denominator of the integrand is larger than D but can be made equal D once the integrand is represented as a derivative, or derivatives, with respect to a coordinate that is not integrated over.

$$
+ \left[-\frac{2\epsilon(1-\epsilon)}{[31]^2[23]^{1-\epsilon}} - \frac{\epsilon(1-\epsilon)}{[31][23]^{2-\epsilon}} + \frac{[12]\epsilon(1-\epsilon)}{[31]^2[23]^{2-\epsilon}} \right] J(2, 2-3\epsilon, \epsilon-1)
$$

$$
+ \frac{1-2\epsilon}{[12]^{1-\epsilon}[31]} J(1+\epsilon, 1-3\epsilon, 2) - \frac{1-2\epsilon}{[12]^{1-\epsilon}[31]^2} J(\epsilon, 1-3\epsilon, 2)
$$

$$
+ \left(\frac{1-\epsilon}{[31][23]^{2-\epsilon}} + \frac{1-\epsilon}{[31]^2[23]^{1-\epsilon}} - \frac{[12](1-\epsilon)}{[31]^2[23]^{2-\epsilon}} \right) J(2, 1-2\epsilon, 0)
$$

$$
- \frac{\epsilon - \epsilon^2}{[31]^2[23]^{1-\epsilon}} J(1, 1-3\epsilon, 1+\epsilon) \Bigg\} \ . \tag{3.1}
$$

Here we use the notation

$$
A(\alpha_1, \alpha_2, \alpha_3) = a(\alpha_1) a(\alpha_2) a(\alpha_2); \qquad a(\alpha) = \frac{\Gamma(D/2 - \alpha)}{\Gamma(\alpha)} \ .
$$

The second and the third integrals (2.5) and (2.6) are given in A, eqs. (A.1) and (A.2), respectively, and have a similar structure as the first integral (3.1). As a consequence, the total Integral I_b can be organized into the sum

$$
I_b = \sum_{k=D-1}^{D+2} I_b^{(k)} , \tag{3.2}
$$

where $I_b^{(k)}$ represents the terms with $J(\alpha_1, \alpha_2, \alpha_3)$ with $\alpha_1 + \alpha_2 + \alpha_3 = k$.

4. Integrals with the sum of indices $D - 1$

The result for $I_b^{(D)}$ and for $I_b^{(D+1)} + I_b^{(D+2)}$ is written in B in terms of D-dimensional J-integrals and then reduced to explicit expressions by using the uniqueness relation and its variants for the integrals. Below we consider the sum $I_b^{(D-1)}$.

$$
I_b^{(D-1)} = \frac{A(1, 1-2\epsilon, 1)}{4\epsilon(1+2\epsilon)(1-2\epsilon)^2} \Bigg\{ \frac{-1 + 5\epsilon - 6\epsilon^2}{[31]^2[12]^{1-\epsilon}} J(\epsilon, 2-3\epsilon, 1)
$$

$$
+ \left[\frac{1 - 7\epsilon + 10\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{1 - 4\epsilon + 5\epsilon^2}{[31][23]^{2-\epsilon}} - \frac{(1 - 4\epsilon + 5\epsilon^2)[21]}{[31]^2[23]^{2-\epsilon}} \right] J(1, 2-3\epsilon, \epsilon)
$$

$$
+ \left[\frac{2 - 6\epsilon + 4\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{2 - 4\epsilon + 2\epsilon^2}{[31][23]^{2-\epsilon}} - \frac{[12](2 - 4\epsilon + 2\epsilon^2)}{[31]^2[23]^{2-\epsilon}} \right] J(2, 1-3\epsilon, \epsilon)
$$

$$
+ \left[\frac{4\epsilon - 4\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{2\epsilon - 2\epsilon^2}{[31][23]^{2-\epsilon}} + \frac{[12](-2\epsilon + 2\epsilon^2)}{[31]^2[23]^{2-\epsilon}} \right] J(2, 2-3\epsilon, \epsilon-1)
$$

$$
+ \frac{-\epsilon + 2\epsilon^2}{[12]^{1-\epsilon}[31]^2} J(\epsilon, 1-3\epsilon, 2)
$$

$$
+ \frac{1 - \epsilon - 2\epsilon^2}{[12]^{1-\epsilon}[31]^2} J(\epsilon-1, 2-3\epsilon, 2)
$$

$$
+ \frac{-2\epsilon + 4\epsilon^2}{[12]^{1-\epsilon}[31]^2} J(\epsilon-1, 3-3\epsilon, 1)
$$

$$
+ \left(\frac{2\epsilon - 6\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{\epsilon - 3\epsilon^2}{[31][23]^{2-\epsilon}} + \frac{(-\epsilon + 3\epsilon^2)[12]}{[31]^2[23]^{2-\epsilon}} \right) J(1, 3-3\epsilon, \epsilon-1) \Bigg\} . \tag{4.1}
$$

The D-dimensional integrals appearing in $I_b^{(D-1)}$ cannot be found by the uniqueness method or its variants. We will reduce them in this section to explicit expressions and to terms proportional to the integrals $J(1,1,1)$, $J(\epsilon, 2-3\epsilon, 1)$, $J(1, 2-3\epsilon, \epsilon)$ and specific derivatives thereof – by applying the integration by parts (IBP) procedure [29, 30]. Using formula (C.1) from C (IBP), we represent integral (4.1) as

$$
\begin{aligned}
I_b^{(D-1)} = {} & \frac{A(1, 1-2\epsilon, 1)}{4\epsilon(1+2\epsilon)(1-2\epsilon)^2} \left\{ \frac{-1 + 4\epsilon - (9/2)\epsilon^2}{[31]^2[12]^{1-\epsilon}} J(\epsilon, 2-3\epsilon, 1) \right. \\
& + \left[\frac{1 - 6\epsilon + (15/2)\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{1 - 7/2\epsilon + 15/4\epsilon^2}{[31][23]^{2-\epsilon}} + \frac{(-1 + 7/2\epsilon - 15/4\epsilon^2)[21]}{[31]^2[23]^{2-\epsilon}} \right] J(1, 2-3\epsilon, \epsilon) \\
& + \left[\frac{2 - 6\epsilon + 4\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{2 - 4\epsilon + 2\epsilon^2}{[31][23]^{2-\epsilon}} - \frac{[12](2 - 4\epsilon + 2\epsilon^2)}{[31]^2[23]^{2-\epsilon}} \right] J(2, 1-3\epsilon, \epsilon) \\
& + \frac{-\epsilon + 2\epsilon^2}{[12]^{1-\epsilon}[31]^2} J(\epsilon, 1-3\epsilon, 2) \\
& + \left[\frac{3\epsilon - (5/2)\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{(3/2)\epsilon - (5/4)\epsilon^2}{[31][23]^{2-\epsilon}} + \frac{[12](-(3/2)\epsilon + (5/4)\epsilon^2)}{[31]^2[23]^{2-\epsilon}} \right] J(2, 2-3\epsilon, \epsilon-1) \\
& \left. + \frac{1 - (5/2)\epsilon^2}{[12]^{1-\epsilon}[31]^2} J(\epsilon-1, 2-3\epsilon, 2) \right\} \\
& + \frac{A(1, 1-2\epsilon, 1)A(1, 1-3\epsilon, 1+\epsilon)}{8\epsilon^2(1+2\epsilon)(1-2\epsilon)^2} \left(\frac{-\epsilon - (1/2)\epsilon^2}{[12]^{2-2\epsilon}[23]^{1-2\epsilon}[31]^{1+2\epsilon}} + \frac{-(\epsilon/2) - (1/4)\epsilon^2}{[12]^{2-2\epsilon}[23]^{2-2\epsilon}[31]^{2\epsilon}} \right. \\
& \left. + \frac{(3/2)\epsilon + (7/4)\epsilon^2}{[12]^{1-2\epsilon}[23]^{2-2\epsilon}[31]^{1+2\epsilon}} \right) .
\end{aligned}
$$

After applying formulas (C.3) and (C.4) (IBP), this formula takes the following form:

$$
\begin{aligned}
I_b^{(D-1)} = {} & \frac{A(1, 1-2\epsilon, 1)}{4\epsilon(1+2\epsilon)(1-2\epsilon)^2} \left\{ \frac{-1 + 5\epsilon - (19/2)\epsilon^2}{[31]^2[12]^{1-\epsilon}} J(\epsilon, 2-3\epsilon, 1) \right. \\
& + \left[\frac{-1 + 6\epsilon - (29/2)\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{-1 + 13/2\epsilon - 41/4\epsilon^2}{[31][23]^{2-\epsilon}} \right. \\
& \left. + \frac{(1 - 13/2\epsilon + 41/4\epsilon^2)[21]}{[31]^2[23]^{2-\epsilon}} \right] J(1, 2-3\epsilon, \epsilon) \\
& + \left[\frac{3\epsilon - (5/2)\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{(3/2)\epsilon - (5/4)\epsilon^2}{[31][23]^{2-\epsilon}} + \frac{[12](-(3/2)\epsilon + (5/4)\epsilon^2)}{[31]^2[23]^{2-\epsilon}} \right] J(2, 2-3\epsilon, \epsilon-1) \\
& \left. + \frac{1 - (5/2)\epsilon^2}{[12]^{1-\epsilon}[31]^2} J(\epsilon-1, 2-3\epsilon, 2) \right\} \\
& - \frac{1}{2} \left[\frac{1}{[31]^2[23]} + \frac{1}{[31][23]^2} - \frac{[12]}{[31]^2[23]^2} \right] J(1, 1, 1) \\
& + \frac{A(1, 1-2\epsilon, 1)A(1, 1-3\epsilon, 1+\epsilon)}{8\epsilon^2(1+2\epsilon)(1-2\epsilon)^2} \left[\frac{-2 + 7\epsilon - 6\epsilon^2}{[12]^{1-2\epsilon}[23]^{1-2\epsilon}[31]^{2+2\epsilon}} + \frac{-2 + (11/2)\epsilon - (1/4)\epsilon^2}{[12]^{1-2\epsilon}[23]^{2-2\epsilon}[31]^{1+2\epsilon}} \right. \\
& \left. + \frac{(2 - 4\epsilon + 2\epsilon^2)}{[12]^{-2\epsilon}[23]^{2-2\epsilon}[31]^{2+2\epsilon}} + \frac{-\epsilon - (1/2)\epsilon^2}{[12]^{2-2\epsilon}[23]^{1-2\epsilon}[31]^{1+2\epsilon}} + \frac{-(\epsilon/2) - (1/4)\epsilon^2}{[12]^{2-2\epsilon}[23]^{2-2\epsilon}[31]^{2\epsilon}} \right] .
\end{aligned}
$$

Finally, applying relations (C.5) derived in C, we obtain the representation

$$
\begin{aligned}
I_b^{(D-1)} = {} & \frac{A(1, 1-2\epsilon, 1)}{4\epsilon(1+2\epsilon)(1-2\epsilon)^2} \left\{ \frac{5\epsilon - 12\epsilon^2}{[31]^2[12]^{1-\epsilon}} J(\epsilon, 2-3\epsilon, 1) \right.\\
& + \left[\frac{-1 + 9\epsilon - 17\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{-1 + 8\epsilon - (23/2)\epsilon^2}{[31][23]^{2-\epsilon}} + \frac{(1 - 8\epsilon + (23/2)\epsilon^2)[21]}{[31]^2[23]^{2-\epsilon}} \right] J(1, 2-3\epsilon, \epsilon) \\
& + \left[\frac{3\epsilon - (5/2)\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{-(3/2)\epsilon + (5/4)\epsilon^2}{[31][23]^{2-\epsilon}} \right.\\
& \quad \left. + \frac{[12](-(3/2)\epsilon + (5/4)\epsilon^2)}{[31]^2[23]^{2-\epsilon}} \right] (13)_\mu \partial_\mu^{(1)} J(1, 2-3\epsilon, \epsilon) \\
& \left. + \frac{1 - (5/2)\epsilon^2}{[12]^{1-\epsilon}[31]^2} (31)_\mu \partial_\mu^{(3)} J(\epsilon, 2-3\epsilon, 1) \right\} \\
& - \frac{1}{2} \left[\frac{1}{[31]^2[23]} + \frac{1}{[31][23]^2} - \frac{[12]}{[31]^2[23]^2} \right] J(1,1,1) \\
& + \frac{A(1, 1-2\epsilon, 1) A(1, 1-3\epsilon, 1+\epsilon)}{8\epsilon^2(1+2\epsilon)(1-2\epsilon)^2} \times \\
& \times \left[\frac{-2 + 7\epsilon - 6\epsilon^2}{[12]^{1-2\epsilon}[23]^{1-2\epsilon}[31]^{2+2\epsilon}} + \frac{-3 + 6\epsilon - 1/2\epsilon^2}{[12]^{1-2\epsilon}[23]^{2-2\epsilon}[31]^{1+2\epsilon}} + \frac{2 - 4\epsilon + 2\epsilon^2}{[12]^{-2\epsilon}[23]^{2-2\epsilon}[31]^{2+2\epsilon}} \right.\\
& \quad \left. + \frac{-4\epsilon - \epsilon^2}{[12]^{2-2\epsilon}[23]^{1-2\epsilon}[31]^{1+2\epsilon}} + \frac{-2\epsilon - \epsilon^2/2}{[12]^{2-2\epsilon}[23]^{2-2\epsilon}[31]^{2\epsilon}} \right] .
\end{aligned} \tag{4.2}
$$

5. The sum

Summing up eqs. (4.2), (B.1) and (B.2), we obtain

$$
\begin{aligned}
I_b = {} & \frac{A(1, 1-2\epsilon, 1)}{4\epsilon(1+2\epsilon)(1-2\epsilon)^2} \left\{ \frac{5\epsilon - 12\epsilon^2}{[31]^2[12]^{1-\epsilon}} J(\epsilon, 2-3\epsilon, 1) \right.\\
& + \left[\frac{-1 + 9\epsilon - 17\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{-1 + 8\epsilon - (23/2)\epsilon^2}{[31][23]^{2-\epsilon}} + \frac{(1 - 8\epsilon + (23/2)\epsilon^2)[21]}{[31]^2[23]^{2-\epsilon}} \right] J(1, 2-3\epsilon, \epsilon) \\
& + \left[\frac{3\epsilon - (5/2)\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{-(3/2)\epsilon + (5/4)\epsilon^2}{[31][23]^{2-\epsilon}} + \frac{[12](-(3/2)\epsilon + (5/4)\epsilon^2)}{[31]^2[23]^{2-\epsilon}} \right] \times \\
& \left. \times (13)_\mu \partial_\mu^{(1)} J(1, 2-3\epsilon, \epsilon) + \frac{1 - (5/2)\epsilon^2}{[12]^{1-\epsilon}[31]^2} (31)_\mu \partial_\mu^{(3)} J(\epsilon, 2-3\epsilon, 1) \right\} \\
& - \frac{1}{2} \left[\frac{1}{[31]^2[23]} + \frac{1}{[31][23]^2} - \frac{[12]}{[31]^2[23]^2} \right] J(1,1,1) \\
& + \frac{1}{8} \left\{ \frac{-19/4}{[12][23][31]^2} + \frac{5}{[12][23]^2[31]} + \frac{1/4}{[12]^2[23][31]} + \frac{-11/4}{[12]^2[23]^2} + \frac{-9/4}{[23]^2[31]^2} + \frac{5/2}{[12]^2[31]^2} \right\} \\
& + \frac{A(1, 1-2\epsilon, 1) A(1, 1-3\epsilon, 1+\epsilon)}{8\epsilon(1+2\epsilon)(1-2\epsilon)^2} \left\{ \frac{-7/2}{[12]^{1-2\epsilon}[23]^{1-2\epsilon}[31]^{2+2\epsilon}} + \frac{-2}{[12]^{1-2\epsilon}[23]^{2-2\epsilon}[31]^{1+2\epsilon}} \right.\\
& \quad + \frac{-1/2}{[12]^{-2\epsilon}[23]^{2-2\epsilon}[31]^{2+2\epsilon}} + \frac{3/2}{[12]^{2-2\epsilon}[23]^{1-2\epsilon}[31]^{1+2\epsilon}} + \frac{3/2}{[12]^{2-2\epsilon}[23]^{2-2\epsilon}[31]^{2\epsilon}} \\
& \quad \left. + \frac{3}{[12]^{2-2\epsilon}[23]^{-2\epsilon}[31]^{2+2\epsilon}} + \frac{-6}{[31]^2[12]^{2-2\epsilon}} + \frac{1}{[31]^2[23]^{2-2\epsilon}} \right\}
\end{aligned}
$$

$$+\frac{A^2(1,1-2\epsilon,1)}{4\epsilon^2(1+2\epsilon)(1-2\epsilon)^2}\left\{\frac{1-\epsilon}{[12]^{1-\epsilon}[23]^{2-\epsilon}[31]}+\frac{1-\epsilon}{[12]^{1-\epsilon}[23]^{1-\epsilon}[31]^2}-\frac{1-\epsilon}{[12]^{-\epsilon}[23]^{2-\epsilon}[31]^2}\right\}.\ (5.1)$$

After applying formulas (C.6) which were obtained by an application of the Gegenbauer polynomial technique (GPT, cf. ref. [35]), we obtain for the terms proportional to $J(\epsilon, 2-3\epsilon, 1)$ and $J(1, 2-3\epsilon, \epsilon)$ in I_b the following explicit result:

$$\begin{aligned}
I_b^{(J)} &\equiv \frac{A(1,1-2\epsilon,1)}{4\epsilon(1+2\epsilon)(1-2\epsilon)^2}\left\{\frac{5\epsilon-12\epsilon^2}{[31]^2[12]^{1-\epsilon}}J(\epsilon,2-3\epsilon,1)\right.\\
&\quad\left.+\left[\frac{-1+9\epsilon-17\epsilon^2}{[31]^2[23]^{1-\epsilon}}+\frac{-1+8\epsilon-(23/2)\epsilon^2}{[31][23]^{2-\epsilon}}+\frac{(1-8\epsilon+(23/2)\epsilon^2)[21]}{[31]^2[23]^{2-\epsilon}}\right]J(1,2-3\epsilon,\epsilon)\right\}\\
&=\frac{A(1,1-2\epsilon,1)A(1-3\epsilon,1,1+\epsilon)}{8\epsilon(1+2\epsilon)(1-2\epsilon)^2}\left[\frac{11}{[12]^{1-\epsilon}[31]^2[23]^{1-\epsilon}}\right.\\
&\quad\left.+\frac{5}{[12]^{1-\epsilon}[31][23]^{2-\epsilon}}+\frac{-5}{[12]^{-\epsilon}[31]^2[23]^{2-\epsilon}}\right]\\
&\quad+\frac{A(1,1-2\epsilon,1)A(1-3\epsilon,1,1+\epsilon)}{8\epsilon^2(1+2\epsilon)(1-2\epsilon)^2}\left[\frac{-1}{[12]^{1-\epsilon}[31]^2[23]^{1-\epsilon}}\right.\\
&\quad\left.+\frac{-1}{[12]^{1-\epsilon}[31][23]^{2-\epsilon}}+\frac{1}{[12]^{-\epsilon}[31]^2[23]^{2-\epsilon}}\right]\\
&\quad+\frac{A(1,1-2\epsilon,1)A(1-3\epsilon,1,1+\epsilon)}{8\epsilon(1+2\epsilon)(1-2\epsilon)^2}\left[\frac{1}{[12]^{1-\epsilon}[31]^2[23]^{1-\epsilon}}\right.\\
&\quad\left.+\frac{1}{[12]^{1-\epsilon}[31][23]^{2-\epsilon}}-\frac{1}{[12]^{-\epsilon}[31]^2[23]^{2-\epsilon}}\right]\ln\frac{[23]}{[12]}\\
&\quad+\frac{1}{8}\left[\frac{4}{[12][31]^2[23]}+\frac{7/2}{[12][31][23]^2}+\frac{-(7/2)}{[31]^2[23]^2}\right]\\
&\quad-\frac{1}{8}\left[\frac{1}{[12][31]^2[23]}+\frac{5}{[12][31][23]^2}+\frac{-5}{[31]^2[23]^2}\right]\ln\frac{[23]}{[12]}\\
&\quad+\frac{1}{16}\left[\frac{1}{[12][31]^2[23]}+\frac{1}{[12][31][23]^2}+\frac{-1}{[31]^2[23]^2}\right]\ln\frac{[12][23]}{[13]^2}\ln\frac{[23]}{[12]}\\
&\quad+\frac{1}{8}\left[\frac{2}{[31]^2[23]}-\frac{[12]}{[31]^2[23]^2}+\frac{1}{[12][23]^2}-\frac{1}{[31]^2[12]}\right]J(1,1,1).\ (5.2)
\end{aligned}$$

The explicit results for the terms $I_b^{(\partial J,1)}, I_b^{(\partial J,2)}$ proportional to derivatives of $J(1, 2-3\epsilon, \epsilon)$ and $J(\epsilon, 2-3\epsilon, 1)$ in I_b are written in D in eqs. (D.1) and (D.2), respectively.

6. The final result for diagram (*b*)

Substitution of expressions (5.2), (D.1) and (D.2) into the sum (5.1), and its expansion in terms of ϵ, is performed in E, eq. (E.2). Performing explicitly the derivatives appearing in eq. (E.2), we obtain the final result for the diagram (*b*):

$$\begin{aligned}
I_b(\epsilon\to 0) = \frac{1}{8}&\left[\frac{-3\ln[23]-4\ln[12]+7\ln[13]}{[12][23][31]^2}+\frac{-8\ln[23]+4\ln[12]+4\ln[13]}{[12][23]^2[31]}\right.\\
&\left.+\frac{3\ln[23]-3\ln[13]}{[12]^2[23][31]}+\frac{5\ln[23]-6\ln[12]+\ln[13]}{[23]^2[31]^2}+\frac{6\ln[23]-6\ln[13]}{[12]^2[31]^2}\right.
\end{aligned}$$

$$+\frac{3\ln[23]-3\ln[13]}{[12]^2[23]^2}\Bigg]$$

$$+\frac{1}{4}\left[\frac{2[12]}{[31]^2[23]^2}+\frac{1}{[12][23]^2}+\frac{-1}{[31]^2[12]}+\frac{-3}{[31][23]^2}\right]J(1,1,1)$$

$$+\frac{1}{8}\left\{\frac{-2}{[12][23][31]^2}+\frac{8}{[12][23]^2[31]}+\frac{-3}{[12]^2[23]^2}+\frac{-3}{[23]^2[31]^2}+\frac{3}{[12]^2[31]^2}\right\}.\,(6.1)$$

The result for the Davydychev integral $J(1,1,1)$ is in ref. [37]. A new integral representation for it has been found in ref. [9]. The term linear in ϵ of the $J(1,1,1)$ integral in $D=4-2\epsilon$ dimensions was obtained in ref. [38], while all-order ϵ-expansion was derived in ref. [39].

7. The final result for *Lcc* correlator

In this section we write the total result for planar two-loop *Lcc* vertex. It contains contribution of five planar diagrams $(a)-(e)$. The first contribution that corresponds to diagram (a) has been found in ref. [9]. We re-present the final formula of ref. [9] in the following form

$$V^{(a)}\equiv\left[\frac{-8}{[12]^2[31]^2}+\frac{8}{[12][23][31]^2}+\frac{16}{[12][23]^2[31]}+\frac{8}{[12]^2[23][31]}\right]$$

$$+\left[\frac{2}{[12]^2[23]}+\frac{2}{[23][31]^2}+\frac{-12}{[12][31]^2}+\frac{-12}{[12]^2[31]}+\frac{-6}{[12][23]^2}+\frac{-6}{[23]^2[31]}\right.$$

$$\left.+\frac{2[12]}{[23]^2[31]^2}+\frac{2[31]}{[12]^2[23]^2}+\frac{8[23]}{[12]^2[31]^2}\right]J(1,1,1)$$

$$+\left[\frac{-6}{[12]^2[23]^2}+\frac{-4}{[12]^2[31]^2}+\frac{2}{[23]^2[31]^2}+\frac{-2}{[12][23][31]^2}\right.$$

$$\left.+\frac{8}{[12][23]^2[31]}+\frac{10}{[12]^2[23][31]}\right]\ln[12]$$

$$+\left[\frac{4}{[12]^2[23]^2}+\frac{8}{[12]^2[31]^2}+\frac{4}{[23]^2[31]^2}+\frac{-8}{[12][23][31]^2}\right.$$

$$\left.+\frac{-16}{[12][23]^2[31]}+\frac{-8}{[12]^2[23][31]}\right]\ln[23]$$

$$+\left[\frac{2}{[12]^2[23]^2}+\frac{-4}{[12]^2[31]^2}+\frac{-6}{[23]^2[31]^2}+\frac{10}{[12][23][31]^2}\right.$$

$$\left.+\frac{8}{[12][23]^2[31]}+\frac{-2}{[12]^2[23][31]}\right]\ln[31]$$
(7.1)

The total contribution of diagram (a) to the *Lcc* correlator is this integral multiplied by a weight factor coming from combinatorics of Feynman rules, trivial group algebra, and Grassmanian nature of the ghost fields. The contributions of all the diagrams $(a),(b),(c)$ to the effective action have common factor ig^4N^2/π^8 in front of them. We concentrate on the relative weights caused by degrees of $1/2$ and signs. This weight factor for diagram (a) is $1/2^{16}$.

The final result for diagram (b) is eq. (6.1). We re-present it in the following form

$$\frac{1}{8}\left[\frac{-3}{[12]^2[23]^2}+\frac{3}{[12]^2[31]^2}+\frac{-3}{[23]^2[31]^2}+\frac{-2}{[12][23][31]^2}+\frac{8}{[12][23]^2[31]}\right]$$

$$+\frac{1}{8}\left[\frac{-2}{[12][31]^2}+\frac{2}{[12][23]^2}+\frac{-6}{[23]^2[31]}+\frac{4[12]}{[23]^2[31]^2}\right]J(1,1,1)$$

$$+\frac{1}{8}\left[\frac{-6}{[23]^2[31]^2}+\frac{-4}{[12][23][31]^2}+\frac{4}{[12][23]^2[31]}\right]\ln[12]$$

$$+\frac{1}{8}\left[\frac{3}{[12]^2[23]^2}+\frac{6}{[12]^2[31]^2}+\frac{5}{[23]^2[31]^2}+\frac{-3}{[12][23][31]^2}\right.$$
$$\left.+\frac{-8}{[12][23]^2[31]}+\frac{3}{[12]^2[23][31]}\right]\ln[23]$$

$$+\frac{1}{8}\left[\frac{-3}{[12]^2[23]^2}+\frac{-6}{[12]^2[31]^2}+\frac{1}{[23]^2[31]^2}+\frac{7}{[12][23][31]^2}\right.$$
$$\left.+\frac{4}{[12][23]^2[31]}+\frac{-3}{[12]^2[23][31]}\right]\ln[31] \tag{7.2}$$

This integral should be summed with another integral obtained from eq. (7.2) by exchanging x_2 and x_3. Thus, the total contribution of diagram (b) to the Lcc correlator at two loop level is

$$V^{(b)} \equiv \frac{1}{8}\left[\frac{-6}{[12]^2[23]^2}+\frac{6}{[12]^2[31]^2}+\frac{-6}{[23]^2[31]^2}+\frac{-2}{[12][23][31]^2}\right.$$
$$\left.+\frac{16}{[12][23]^2[31]}+\frac{-2}{[12]^2[23][31]}\right]+\frac{1}{8}\left[\frac{-2}{[12][31]^2}+\frac{-2}{[12]^2[31]}+\frac{-4}{[12][23]^2}+\frac{-4}{[23]^2[31]}\right.$$
$$\left.+\frac{4[12]}{[23]^2[31]^2}+\frac{4[31]}{[23]^2[31]^2}\right]J(1,1,1)$$

$$+\frac{1}{8}\left[\frac{1}{[12]^2[23]^2}+\frac{-6}{[12]^2[31]^2}+\frac{-9}{[23]^2[31]^2}+\frac{-7}{[12][23][31]^2}\right.$$
$$\left.+\frac{8}{[12][23]^2[31]}+\frac{7}{[12]^2[23][31]}\right]\ln[12]$$

$$+\frac{1}{8}\left[\frac{8}{[12]^2[23]^2}+\frac{12}{[12]^2[31]^2}+\frac{8}{[23]^2[31]^2}+\frac{-16}{[12][23]^2[31]}\right]\ln[23]$$

$$+\frac{1}{8}\left[\frac{-9}{[12]^2[23]^2}+\frac{-6}{[12]^2[31]^2}+\frac{1}{[23]^2[31]^2}+\frac{7}{[12][23][31]^2}\right.$$
$$\left.+\frac{8}{[12][23]^2[31]}+\frac{-7}{[12]^2[23][31]}\right]\ln[31] \tag{7.3}$$

The weight of diagram (b) in the full result is $-1/2^{13}$.

The result for diagram (c) is obtained in a different way than for diagrams (a) and (b) and the details of calculation of the diagram (c) can be found in ref. [59]. It takes the form

$$\left[\frac{13/2}{[12]^2[23]^2}+\frac{-33/2}{[12]^2[31]^2}+\frac{15/2}{[23]^2[31]^2}+\frac{11}{[12][23][31]^2}+\frac{-14}{[12][23]^2[31]}+\frac{10}{[12]^2[23][31]}\right]$$

$$+\left[\frac{-6}{[12][31]^2}+\frac{-2}{[12]^2[31]}+\frac{6}{[12][23]^2}+\frac{-5}{[23]^2[31]}+\frac{[12]}{[23]^2[31]^2}\right.$$
$$\left.+\frac{-2[31]}{[12]^2[23]^2}+\frac{4[23]}{[12]^2[31]^2}\right]J(1,1,1)$$

$$+\left[\frac{-2}{[12]^2[23]^2}+\frac{6}{[12]^2[31]^2}+\frac{4}{[23]^2[31]^2}+\frac{1}{[12][23][31]^2}\right.$$

$$+ \frac{-1}{[12][23]^2[31]} + \frac{-4}{[12]^2[23][31]} \Bigg] \ln[12]$$

$$+ \Bigg[\frac{-1/2}{[12]^2[23]^2} + \frac{-5}{[12]^2[31]^2} + \frac{-3/2}{[23]^2[31]^2} + \frac{-7/2}{[12][23][31]^2}$$

$$+ \frac{2}{[12][23]^2[31]} + \frac{7/2}{[12]^2[23][31]} \Bigg] \ln[23]$$

$$+ \Bigg[\frac{5/2}{[12]^2[23]^2} + \frac{-1}{[12]^2[31]^2} + \frac{-5/2}{[23]^2[31]^2} + \frac{5/2}{[12][23][31]^2}$$

$$+ \frac{-1}{[12][23]^2[31]} + \frac{1/2}{[12]^2[23][31]} \Bigg] \ln[31] \tag{7.4}$$

This integral should be summed with another integral obtained from eq. (7.4) by exchanging x_2 and x_3. Thus, the total contribution of diagram (c) is

$$V^{(c)} \equiv \Bigg[\frac{14}{[12]^2[23]^2} + \frac{-33}{[12]^2[31]^2} + \frac{14}{[23]^2[31]^2} + \frac{21}{[12][23][31]^2}$$

$$+ \frac{-28}{[12][23]^2[31]} + \frac{21}{[12]^2[23][31]} \Bigg] + \Bigg[\frac{-8}{[12][31]^2} + \frac{-8}{[12]^2[31]} + \frac{1}{[12][23]^2} + \frac{1}{[23]^2[31]}$$

$$+ \frac{-[12]}{[23]^2[31]^2} + \frac{-[31]}{[12]^2[23]^2} + \frac{8[23]}{[12]^2[31]^2} \Bigg] J(1,1,1)$$

$$+ \Bigg[\frac{-9/2}{[12]^2[23]^2} + \frac{5}{[12]^2[31]^2} + \frac{13/2}{[23]^2[31]^2} + \frac{3/2}{[12][23][31]^2}$$

$$+ \frac{-2}{[12][23]^2[31]} + \frac{-3/2}{[12]^2[23][31]} \Bigg] \ln[12]$$

$$+ \Bigg[\frac{-2}{[12]^2[23]^2} + \frac{-10}{[12]^2[31]^2} + \frac{-2}{[23]^2[31]^2} + \frac{4}{[12][23]^2[31]} \Bigg] \ln[23]$$

$$+ \Bigg[\frac{13/2}{[12]^2[23]^2} + \frac{5}{[12]^2[31]^2} + \frac{-9/2}{[23]^2[31]^2} + \frac{-3/2}{[12][23][31]^2}$$

$$+ \frac{-2}{[12][23]^2[31]} + \frac{3/2}{[12]^2[23][31]} \Bigg] \ln[31] \tag{7.5}$$

The weight of diagram (c) in the full result is $1/2^{15}$.

Thus, the contribution of first three diagrams $(a), (b)$ and (c) to the two-loop effective action can be presented in the following form

$$\int d^4x_1 d^4x_2 d^4x_3 \frac{ig^4 N^2}{2^{16}\pi^8} f^{def} L^d(x_1) c^e(x_2) c^f(x_3) \ V^{(a+b+c)}(x_1, x_2, x_3),$$

where

$$V^{(a+b+c)}(x_1, x_2, x_3) \equiv V^{(a)} - 8V^{(b)} + 2V^{(c)} =$$

$$\Bigg[\frac{34}{[12]^2[23]^2} + \frac{-80}{[12]^2[31]^2} + \frac{34}{[23]^2[31]^2} + \frac{52}{[12][23][31]^2} + \frac{-56}{[12][23]^2[31]} + \frac{52}{[12]^2[23][31]} \Bigg]$$

$$+ \Bigg[\frac{2}{[12]^2[23]} + \frac{2}{[23][31]^2} + \frac{-26}{[12][31]^2} + \frac{-26}{[12]^2[31]}$$

$$+ \frac{-4[12]}{[23]^2[31]^2} + \frac{-4[31]}{[12]^2[23]^2} + \frac{24[23]}{[12]^2[31]^2} \Bigg] J(1,1,1)$$

$$+ \left[\frac{-16}{[12]^2[23]^2} + \frac{12}{[12]^2[31]^2} + \frac{24}{[23]^2[31]^2} + \frac{8}{[12][23][31]^2} + \frac{-4}{[12][23]^2[31]} \right] \ln[12]$$

$$+ \left[\frac{-8}{[12]^2[23]^2} + \frac{-24}{[12]^2[31]^2} + \frac{-8}{[23]^2[31]^2} \right.$$

$$\left. + \frac{-8}{[12][23][31]^2} + \frac{8}{[12][23]^2[31]} + \frac{-8}{[12]^2[23][31]} \right] \ln[23]$$

$$+ \left[\frac{24}{[12]^2[23]^2} + \frac{12}{[12]^2[31]^2} + \frac{-16}{[23]^2[31]^2} + \frac{-4}{[12][23]^2[31]} + \frac{8}{[12]^2[23][31]} \right] \ln[31]$$

Diagrams (d) and (e) are easy to calculate since they contain propagator-like insertions only. They can produce logarithms only (due to the shift of the indices in the propagators) and cannot produce Davydychev integral $J(1,1,1)$. These two diagrams are divergent separately in the ultraviolet region, however in the maximally supersymmetric Yang-Mills theory their poles cancel each other. Thus, the contribution of diagrams (d) and (e) to the two-loop effective action can be presented in the following form

$$\int d^4x_1 d^4x_2 d^4x_3 \frac{ig^4N^2}{2^{16}\pi^8} f^{abc} L^a(x_1) c^b(x_2) c^c(x_3) \, V^{(d+e)}(x_1,x_2,x_3),$$

where

$$V^{(d+e)}(x_1,x_2,x_3) \equiv$$

$$\left[\frac{-44/3}{[12]^2[23]^2} + \frac{304/3}{[12]^2[31]^2} + \frac{-44/3}{[23]^2[31]^2} + \frac{-260/3}{[12][23][31]^2} + \frac{88/3}{[12][23]^2[31]} + \frac{-260/3}{[12]^2[23][31]} \right]$$

$$\left[\frac{12}{[12]^2[23]^2} + \frac{-24}{[12]^2[31]^2} + \frac{12}{[23]^2[31]^2} + \frac{12}{[12][23][31]^2} \right.$$

$$\left. + \frac{-24}{[12][23]^2[31]} + \frac{12}{[12]^2[23][31]} \right] \ln[12]$$

$$+ \left[\frac{-24}{[12]^2[23]^2} + \frac{48}{[12]^2[31]^2} + \frac{-24}{[23]^2[31]^2} + \frac{-24}{[12][23][31]^2} \right.$$

$$\left. + \frac{48}{[12][23]^2[31]} + \frac{-24}{[12]^2[23][31]} \right] \ln[23]$$

$$+ \left[\frac{12}{[12]^2[23]^2} + \frac{-24}{[12]^2[31]^2} + \frac{12}{[23]^2[31]^2} + \frac{12}{[12][23][31]^2} \right.$$

$$\left. + \frac{-24}{[12][23]^2[31]} + \frac{12}{[12]^2[23][31]} \right] \ln[31] \tag{7.6}$$

Thus, the full result for the two-loop Lcc correlator in the maximally supersymmetric Yang-Mills theory in four spacetime dimensions is a combination of $V^{(a)}$, $V^{(b)}$, $V^{(c)}$ and $V^{(d+e)}$ with the corresponding weights and has the form

$$\int d^4x_1 d^4x_2 d^4x_3 \frac{ig^4N^2}{2^{15}\pi^8} f^{abc} L^a(x_1) c^b(x_2) c^c(x_3) \, V^{(2)}(x_1,x_2,x_3),$$

where

$$V^{(2)}(x_1, x_2, x_3) = V^{(a+b+c)}(x_1, x_2, x_3) + V^{(d+e)}(x_1, x_2, x_3) =$$
$$\left[\frac{29/3}{[12]^2[23]^2} + \frac{32/3}{[12]^2[31]^2} + \frac{29/3}{[23]^2[31]^2} + \frac{-52/3}{[12][23][31]^2} + \frac{-40/3}{[12][23]^2[31]} + \frac{-52/3}{[12]^2[23][31]} \right]$$
$$+ \left[\frac{1}{[12]^2[23]} + \frac{1}{[23][31]^2} + \frac{-13}{[12][31]^2} + \frac{-13}{[12]^2[31]} + \frac{-2[12]}{[23]^2[31]^2} \right.$$
$$\left. + \frac{-2[31]}{[12]^2[23]^2} + \frac{12[23]}{[12]^2[31]^2} \right] J(1,1,1)$$
$$+ \left[\frac{-2}{[12]^2[23]^2} + \frac{-6}{[12]^2[31]^2} + \frac{18}{[23]^2[31]^2} + \frac{10}{[12][23][31]^2} \right.$$
$$\left. + \frac{-14}{[12][23]^2[31]} + \frac{6}{[12]^2[23][31]} \right] \ln[12]$$
$$+ \left[\frac{-16}{[12]^2[23]^2} + \frac{12}{[12]^2[31]^2} + \frac{-16}{[23]^2[31]^2} + \frac{-16}{[12][23][31]^2} \right.$$
$$\left. + \frac{28}{[12][23]^2[31]} + \frac{-16}{[12]^2[23][31]} \right] \ln[23]$$
$$+ \left[\frac{18}{[12]^2[23]^2} + \frac{-6}{[12]^2[31]^2} + \frac{-2}{[23]^2[31]^2} + \frac{6}{[12][23][31]^2} \right.$$
$$\left. + \frac{-14}{[12][23]^2[31]} + \frac{10}{[12]^2[23][31]} \right] \ln[31]$$

For the reference, the one-loop contribution to the effective action is

$$\int d^4x_1 d^4x_2 d^4x_3 \frac{ig^2 N}{2^8 \pi^6} f^{abc} L^a(x_1) c^b(x_2) c^c(x_3) \ V^{(1)}(x_1, x_2, x_3),$$

where

$$V^{(1)}(x_1, x_2, x_3) =$$
$$\left[\frac{-1}{[12]^2[23]^2} + \frac{2}{[12]^2[31]^2} + \frac{-1}{[23]^2[31]^2} + \frac{-1}{[12][23][31]^2} + \frac{2}{[12][23]^2[31]} + \frac{-1}{[12]^2[23][31]} \right]$$

8. Conclusions

In this paper we presented the calculation of the second diagram [diagram (b)] of the five planar two-loop diagrams to the Lcc vertex depicted in figure 1 for a general Yang-Mills theory. The Lorentz structure of the corresponding integrand is complicated in comparison with the integrand of the previously calculated diagram (a) [9]. In order to apply the same methods as in the diagram (a), we first performed in the integrands the Lorentz algebra for certain subproducts in the numerators, then performed one of the two D-dimensional integrations, then finished the Lorentz algebra, and finally performed the second D-dimensional integration. It is a long procedure that requires certain computer resources. In principle, it is possible to use another trick to reduce the calculation [59], a trick which we applied

in the calculation of diagram (c) [59]. Neither the result for the diagram (b) obtained here, nor the result for the diagram (a) obtained in our previous work [9] depend on any scale, infrared or ultraviolet, in complete correspondence with the naive index counting arguments, taking into account that none of three diagrams (a, b, c) contains any divergent subgraphs due to transversality of the gauge propagator in the Landau gauge. From the point of view of R-operation theory the scale independence of this vertex in the Landau gauge in all orders of perturbation theory has been explained in refs. [1−3]. By using the ST identity, other (leading-N) two-loop vertices can be derived from the (leading-N) two-loop Lcc vertex once the latter is known. For example, it will be possible to derive the four-point off-shell correlator, and consequently reproduce the known result for the four-point gluon amplitude [18, 19] and the anomalous dimensions of twist-two operators [40−44].

It is possible to look at these results from different points of view. On the one hand, singular parts of the diagrams (d) and (e) are proportional to the one-loop result for the Lcc correlator written in ref. [9]. Moreover, the transversality of the gauge propagator must be conserved by the radiative correction, since it is a well-known fact that the gauge fixing term does not obtain any quantum correction [15]. This means that the sum of singular parts of diagrams (d) and (e) is proportional to the one-loop contribution, with a coefficient that is singular in a general nonsupersymmetric massless gauge theory, but in this particular theory it is a finite factor since the poles in ϵ cancel in the sum of the diagrams (d) and (e) due to the $\mathcal{N} = 4$ supersymmetry. This factor is logarithm of a ratio of the spacetime intervals. This factor appears due to shift of indices by multiplies of ϵ in the propagator of the fields. This shift is typical for massless theories in MS scheme. In the nonsupersymmetric case, that singular number can be absorbed into the gauge coupling to organize the bare coupling. Then, the bare coupling together with the logarithm of ratio of the distance to the scale, leads to the running (effective) coupling. It means that in D dimensions the massless nonsupersymmetric gauge theory is a conformal gauge theory in terms of the running effective coupling (formed from the bare coupling) and dressed mean fields. In other words, we think that, in terms of the running couplings and dressed mean fields, the arguments of ref. [1] can be applied without modifications to the massless QCD. The role of the renormalization group (RG) scale could be the coordinate of the moduli space of the theory.

On the other hand, QCD is asymptotically free theory and for short distances we can consider it as a theory with zero beta-function (running of the effective charge is almost absent) and the conformal structure could be restored at short distances by the method of conformal theory in terms of dressed mean fields. Further, it would be interesting to investigate the relation of off-shell correlators of dressed mean fields and correlators of instantons in multi-dimensional theories. To introduce masses in the theory, we need to use softly broken supersymmetry, in which the couplings are spacetime-independent background superfields. The relation between the RG functions of softly broken and rigid theories was found out in refs. [45−47]. The relation between the correlators of softly broken and rigid theories can be found by a trick of general change of variables in superspace [48].

To restore the conformal structure of all the effective action, we need to solve the ST identity. Algorithm for solving the ST identity could be applied to various theories, such

as the $\mathcal{N} = 8$ supergravity [49–52], Chern-Simons theory near the RG fixed points [53], massless gauge theory near fixed points in the coupling space, topological field theories in higher dimensions, finite $\mathcal{N} = 1$ supersymmetric theories [54–58]. We further note that, in addition to the five planar two-loop diagrams of figure 1, there is one nonplanar diagram (the nonplanar variant of the diagram (a)) which is suppressed in the planar limit of the large 't Hooft coupling by the factor $1/N$.

Acknowledgments

Note added. After publication of the first version of this paper the "dual conformal symmetry" of the momentum space (conformal symmetry of four-point correlator of gluon in the momentum space) found in ref. [20] has been exploited further in ref. [60] up to four loop level to detect all off-shell correlators that will contribute to four-point amplitude. That symmetry appears also on the strong coupling side as it has been found by Alday and Maldacena in ref. [61] via AdS/CFT correspondence. As we have already mentioned in Introduction, Fourier transform of the UD integrals of the momentum space can be expressed in terms of UD integrals in the position space. This observation could be a bridge between the dual conformal symmetry of the momentum space and conformal symmetry of the effective action of dressed mean fields in the position space.

A. Reduction of $I_{b,2}$ and $I_{b,3}$ to single scalar integrals

In this appendix we present the result for the second and the third integrals (2.5) and (2.6) in terms of D-dimensional integrals J. The result for the second integral is

$$
\begin{aligned}
I_{b,2} = \frac{A(1, 1-2\epsilon, 1)}{8\epsilon(1-2\epsilon)^2} & \left\{ \frac{(8-20\epsilon)(1-\epsilon^2)}{[31][12]^{-\epsilon}} J(2+\epsilon, 2-3\epsilon, 1) \right. \\
& + \frac{(1-\epsilon^2)(4-6\epsilon)}{[31][12]^{-1-\epsilon}} J(2+\epsilon, 3-3\epsilon, 1) \\
& + \left[-\frac{2\epsilon^2(1-\epsilon)}{[31][12]^{1-\epsilon}} + \frac{2(2-\epsilon-4\epsilon^2)(1-\epsilon)}{[31]^2[12]^{-\epsilon}} \right] J(\epsilon, 3-3\epsilon, 1) \\
& \left. + \frac{2(1-\epsilon^2)(2-3\epsilon)}{[31][12]^{1-\epsilon}} J(2+\epsilon, 1-3\epsilon, 1) \right. \\
& - \frac{2(2-\epsilon-4\epsilon^2)(1-\epsilon)}{[31]^2[12]^{1-\epsilon}} J(1+\epsilon, 2-3\epsilon, 0) + \frac{2(1-\epsilon^2)(2-3\epsilon)}{[31]^2[12]^{1-\epsilon}} J(2+\epsilon, 1-3\epsilon, 0) \\
& \left. - \frac{2\epsilon^2(1-\epsilon)}{[31]^2[12]^{1-\epsilon}} J(\epsilon, 3-3\epsilon, 0) \right.
\end{aligned}
$$

$$
+ \left[-\frac{7 - 17\epsilon - 8\epsilon^2 + 20\epsilon^3}{[31]^2[12]^{-\epsilon}} - \frac{2(2 - \epsilon - 4\epsilon^2)(1 - \epsilon)}{[31][12]^{1-\epsilon}} \right] J(1 + \epsilon, 2 - 3\epsilon, 1)
$$

$$
+ \left[-\frac{2(1 - \epsilon^2)(2 - 3\epsilon)}{[31]^2[12]^{-1-\epsilon}} - \frac{2(2 - \epsilon - 4\epsilon^2)(1 - \epsilon)}{[31][12]^{-\epsilon}} \right] J(1 + \epsilon, 3 - 3\epsilon, 1)
$$

$$
+ \frac{4 - 5\epsilon - 6\epsilon^2 + 8\epsilon^3}{[31]^2[12]^{1-\epsilon}} J(\epsilon, 2 - 3\epsilon, 1)
$$

$$
+ \frac{2\epsilon^2(1 - \epsilon)}{[31]^2[12]^{1-\epsilon}} J(\epsilon - 1, 3 - 3\epsilon, 1) - \frac{(1 - 2\epsilon^2)(2 - 3\epsilon)}{[31]^2[12]^{1-\epsilon}} J(1 + \epsilon, 1 - 3\epsilon, 1)
$$

$$
+ \left(\frac{\epsilon(2 - \epsilon)[21]}{[31][23]^{1-\epsilon}} - \frac{(2 - \epsilon)(2 - 3\epsilon)}{[31][23]^{-\epsilon}} \right) J(2, 2 - 3\epsilon, 1 + \epsilon)
$$

$$
+ \left(\frac{(\epsilon^2 - 2\epsilon)[21]}{[31]^2[23]^{1-\epsilon}} + \frac{4 - 8\epsilon + 3\epsilon^2}{[31]^2[23]^{-\epsilon}} + \frac{\epsilon^2 - \epsilon}{[31][23]^{1-\epsilon}} \right) J(1, 2 - 3\epsilon, 1 + \epsilon)
$$

$$
+ \left(\frac{4 - 10\epsilon + 6\epsilon^2}{[31]^2[23]^{-\epsilon}} - \frac{\epsilon(1 - 2\epsilon)}{[31][23]^{1-\epsilon}} + \frac{(3\epsilon - 4\epsilon^2)[21]}{[31]^2[23]^{1-\epsilon}} \right) J(1, 3 - 3\epsilon, \epsilon)
$$

$$
- \frac{(2 - \epsilon)(2 - 3\epsilon)[21]}{[31][23]^{-\epsilon}} J(2, 3 - 3\epsilon, 1 + \epsilon)
$$

$$
+ \left(\frac{(2 - \epsilon)(2 - 3\epsilon)[21]}{[31]^2[23]^{-\epsilon}} + \frac{(2 - \epsilon)(2 - 3\epsilon)}{[31][23]^{-\epsilon}} \right) J(1, 3 - 3\epsilon, 1 + \epsilon)
$$

$$
+ \left[\frac{\epsilon(1 + \epsilon)[21]}{[31]^2[23]^{1-\epsilon}} + \frac{\epsilon(1 - \epsilon)[21]}{[31][23]^{2-\epsilon}} - \frac{[12]^2\epsilon(1 - \epsilon)}{[31]^2[23]^{2-\epsilon}} \right.
$$
$$
\left. - \frac{2(1 - \epsilon)(2 - 3\epsilon)}{[31]^2[23]^{-\epsilon}} + \frac{\epsilon(1 - 2\epsilon)}{[31][23]^{1-\epsilon}} \right] J(2, 2 - 3\epsilon, \epsilon)
$$

$$
+ \left[-\frac{2\epsilon(1 - \epsilon)[21]}{[31]^2[23]^{1-\epsilon}} - \frac{\epsilon(1 - \epsilon)[21]}{[31][23]^{2-\epsilon}} + \frac{[12]^2\epsilon(1 - \epsilon)}{[31]^2[23]^{2-\epsilon}} \right] J(2, 3 - 3\epsilon, \epsilon - 1)
$$

$$
+ \left[-\frac{2(2 - 3\epsilon)(1 - \epsilon)[21]}{[31]^2[23]^{-\epsilon}} + \frac{\epsilon(1 - 2\epsilon)[21]}{[31][23]^{1-\epsilon}} - \frac{[12]^2\epsilon(2 - 3\epsilon)}{[31]^2[23]^{1-\epsilon}} \right] J(2, 3 - 3\epsilon, \epsilon)
$$

$$
+ \frac{1 - 3\epsilon}{[21]^{-\epsilon}[31]} J(1 + \epsilon, 2 - 3\epsilon, 2) + \left(-\frac{1 - 3\epsilon}{[31]^2[21]^{-\epsilon}} + \frac{\epsilon}{[31][21]^{1-\epsilon}} \right) J(\epsilon, 2 - 3\epsilon, 2)
$$

$$
- \frac{\epsilon}{[31]^2[21]^{1-\epsilon}} J(\epsilon - 1, 2 - 3\epsilon, 2)
$$

$$
+ \frac{2 - 3\epsilon}{[31][21]^{1-\epsilon}} J(1 + \epsilon, 1 - 3\epsilon, 2) - \frac{2 - 3\epsilon}{[31]^2[21]^{1-\epsilon}} J(\epsilon, 1 - 3\epsilon, 2)
$$

$$
+ \frac{\epsilon(1 - \epsilon)}{[31][23]^{1-\epsilon}} J(2, 1 - 3\epsilon, 1 + \epsilon)
$$

$$
+ \left(-\frac{(1 - 2\epsilon)(1 - \epsilon)}{[31]^2[23]^{1-\epsilon}} - \frac{(1 - \epsilon)^2}{[31][23]^{2-\epsilon}} + \frac{(1 - \epsilon)^2[12]}{[31]^2[23]^{2-\epsilon}} \right) J(2, 1 - 3\epsilon, \epsilon)
$$

$$
- \frac{\epsilon(1 - \epsilon)}{[31]^2[23]^{1-\epsilon}} J(1, 1 - 3\epsilon, 1 + \epsilon)
$$

$$
+ \left(-\frac{2\epsilon(1 - \epsilon)}{[31]^2[23]^{1-\epsilon}} - \frac{\epsilon(1 - \epsilon)}{[31][23]^{2-\epsilon}} + \frac{\epsilon(1 - \epsilon)[12]}{[31]^2[23]^{2-\epsilon}} \right) J(2, 2 - 3\epsilon, \epsilon - 1)
$$

$$
+ \left(\frac{(1 - \epsilon)^2}{[31]^2[23]^{1-\epsilon}} + \frac{(1 - \epsilon)^2}{[31][23]^{2-\epsilon}} - \frac{(1 - \epsilon)^2[21]}{[31]^2[23]^{2-\epsilon}} \right) J(1, 2 - 3\epsilon, \epsilon)
$$

$$+ \frac{\epsilon(1-\epsilon)}{[31]^2[23]^{1-\epsilon}} J(0, 2-3\epsilon, 1+\epsilon)$$

$$+ \left(\frac{2\epsilon(1-\epsilon)}{[31]^2[23]^{1-\epsilon}} + \frac{\epsilon(1-\epsilon)}{[31][23]^{2-\epsilon}} - \frac{\epsilon(1-\epsilon)[12]}{[31]^2[23]^{2-\epsilon}} \right) J(1, 3-3\epsilon, \epsilon-1)$$

$$- \frac{\epsilon(1-\epsilon)}{[31]^2[23]^{1-\epsilon}} J(0, 3-3\epsilon, \epsilon)$$

$$+ \left(\frac{1-\epsilon}{[31][23]^{2-\epsilon}} + \frac{1-\epsilon}{[31]^2[23]^{1-\epsilon}} - \frac{(1-\epsilon)[12]}{[31]^2[23]^{2-\epsilon}} \right) J(2, 1-2\epsilon, 0)$$

$$- \left. \left(\frac{1-\epsilon}{[31][23]^{2-\epsilon}} + \frac{1-\epsilon}{[31]^2[23]^{1-\epsilon}} - \frac{(1-\epsilon)[12]}{[31]^2[23]^{2-\epsilon}} \right) J(1, 2-2\epsilon, 0) \right\} . \qquad (A.1)$$

The third double integral (2.6) is simpler, the integration over y can be taken immediately,

$$I_{b,3} = \frac{A(1, 1-2\epsilon, 1)}{4\epsilon(1-2\epsilon)} \left\{ - \frac{1-2\epsilon^2}{[12]^{1-\epsilon}[31]^2} J(1+\epsilon, 1-3\epsilon, 1) \right.$$

$$\left(- \frac{1+\epsilon-4\epsilon^2}{[12]^{-\epsilon}[31]^2} + \frac{(2\epsilon-2\epsilon^2)[23]}{[12]^{1-\epsilon}[31]^2} + \frac{1-2\epsilon-2\epsilon^2}{[12]^{1-\epsilon}[31]} \right) J(1+\epsilon, 2-3\epsilon, 1)$$

$$+ \frac{1+2\epsilon-4\epsilon^2}{[12]^{1-\epsilon}[31]^2} J(\epsilon, 2-3\epsilon, 1) - \frac{2+2\epsilon-4\epsilon^2}{[12]^{1-\epsilon}[31]^2} J(1+\epsilon, 2-3\epsilon, 0)$$

$$- \frac{2\epsilon-2\epsilon^2}{[12]^{-\epsilon}[31]^2} J(1+\epsilon, 3-3\epsilon, 0) + \frac{2\epsilon-2\epsilon^2}{[12]^{1-\epsilon}[31]^2} J(\epsilon, 3-3\epsilon, 0)$$

$$+ \left(\frac{(2\epsilon-2\epsilon^2)[23]}{[12]^{-\epsilon}[31]^2} - \frac{\epsilon}{[12]^{-\epsilon}[31]} - \frac{\epsilon-2\epsilon^2}{[12]^{-\epsilon-1}[31]^2} \right) J(1+\epsilon, 3-3\epsilon, 1)$$

$$+ \frac{2+\epsilon}{[12]^{1-\epsilon}[31]} J(1+\epsilon, 1-3\epsilon, 2)$$

$$+ \left(- \frac{2+\epsilon}{[12]^{1-\epsilon}[31]} - \frac{\epsilon[23]}{[12]^{1-\epsilon}[31]^2} - \frac{1}{[12]^{-\epsilon}[31]^2} \right) J(\epsilon, 2-3\epsilon, 2)$$

$$+ \left(- \frac{2\epsilon(1-\epsilon)[23]}{[12]^{1-\epsilon}[31]^2} + \frac{\epsilon}{[12]^{1-\epsilon}[31]} + \frac{3\epsilon-4\epsilon^2}{[12]^{-\epsilon}[31]^2} \right) J(\epsilon, 3-3\epsilon, 1)$$

$$+ \frac{\epsilon[23]}{[12]^{-\epsilon}[31]} J(1+\epsilon, 3-3\epsilon, 2) - \left(\frac{\epsilon[23]}{[12]^{1-\epsilon}[31]} + \frac{\epsilon[23]}{[12]^{-\epsilon}[31]^2} \right) J(\epsilon, 3-3\epsilon, 2)$$

$$- \frac{1}{[12]^{1-\epsilon}[31]^2} J(\epsilon, 1-3\epsilon, 2) + \frac{1}{[12]^{1-\epsilon}[31]^2} J(\epsilon-1, 2-3\epsilon, 2)$$

$$- \frac{2\epsilon-2\epsilon^2}{[12]^{1-\epsilon}[31]^2} J(\epsilon-1, 3-3\epsilon, 1) + \frac{\epsilon[23]}{[12]^{1-\epsilon}[31]^2} J(\epsilon-1, 3-3\epsilon, 2)$$

$$+ \frac{2-2\epsilon^2}{[12]^{1-\epsilon}[31]^2} J(2+\epsilon, 1-3\epsilon, 0)$$

$$- \frac{1+\epsilon}{[12]^{1-\epsilon}} J(2+\epsilon, 1-3\epsilon, 2) - \frac{1+\epsilon}{[12]^{-\epsilon}} J(2+\epsilon, 2-3\epsilon, 2)$$

$$+ \left(\frac{1+\epsilon}{[12]^{1-\epsilon}} + \frac{2+\epsilon}{[12]^{-\epsilon}[31]} + \frac{\epsilon[23]}{[12]^{1-\epsilon}[31]} \right) J(1+\epsilon, 2-3\epsilon, 2)$$

$$- \left. \frac{1-\epsilon-2\epsilon^2}{[12]^{1-\epsilon}[31]} J(2+\epsilon, 1-3\epsilon, 1) - \frac{1-\epsilon-2\epsilon^2}{[12]^{-\epsilon}[31]} J(2+\epsilon, 2-3\epsilon, 1) \right\} . \qquad (A.2)$$

In eqs. (A.1) and (A.2), we do not write single integrals with one of the indices zero and sum of other two indices equal to D, $D+1$ or $D+2$. Such terms are proportional to delta-functions in position space multiplied by pole in ϵ and will disappear in the final expression (6.1).

B. $I_b^{(D)}$, $I_b^{(D+1)}$ and $I_b^{(D+2)}$ contributions to I_b

In this appendix we write the results for the terms contributing to the diagram (b) with J's whose sum of indices is $k = D, D+1, D+2$. These terms appear in I_b which is the sum of eqs. (3.1), (A.1) and (A.2). The integrals in $I_b^{(D)}$ are calculated by using the uniqueness method

$$
\begin{aligned}
I_b^{(D)} = \frac{A(1, 1-2\epsilon, 1)}{4\epsilon(1+2\epsilon)(1-2\epsilon)^2} &\left\{ \left(\frac{10 - 18\epsilon - 12\epsilon^2}{[31]^2[12]^{-\epsilon}} + \frac{3 - 4\epsilon - 10\epsilon^2}{[31][12]^{1-\epsilon}} \right.\right. \\
&\left. + \frac{(2\epsilon - 2\epsilon^2)[23]}{[31]^2[12]^{1-\epsilon}} \right) J(1+\epsilon, 2-3\epsilon, 1) \\
&+ \frac{-3 + 3\epsilon + 12\epsilon^2}{[31][12]^{1-\epsilon}} J(2+\epsilon, 1-3\epsilon, 1) + \frac{2\epsilon - 4\epsilon^2}{[31]^2[12]^{1-\epsilon}} J(2+\epsilon, 1-3\epsilon, 0) \\
&+ \frac{-4\epsilon + 8\epsilon^2}{[31]^2[12]^{1-\epsilon}} J(1+\epsilon, 2-3\epsilon, 0) + \frac{-2\epsilon + 2\epsilon^2}{[31][23]^{1-\epsilon}} J(2, 1-3\epsilon, 1+\epsilon) \\
&+ \left[\frac{-8 + 16\epsilon - 6\epsilon^2}{[31]^2[23]^{-\epsilon}} + \frac{-\epsilon + 3\epsilon^2}{[31][23]^{1-\epsilon}} + \frac{(-2\epsilon + 5\epsilon^2)[21]}{[31]^2[23]^{1-\epsilon}} \right] J(1, 2-3\epsilon, 1+\epsilon) \\
&+ \left[\frac{(7\epsilon - 14\epsilon^2)[21]}{[31]^2[23]^{1-\epsilon}} + \frac{(\epsilon - 3\epsilon^2)[21]}{[31][23]^{2-\epsilon}} + \frac{[12]^2(-\epsilon + 3\epsilon^2)}{[31]^2[23]^{2-\epsilon}} \right. \\
&\left. + \frac{8 - 20\epsilon + 12\epsilon^2}{[31]^2[23]^{-\epsilon}} + \frac{-2\epsilon + 4\epsilon^2)}{[31][23]^{1-\epsilon}} \right] J(2, 2-3\epsilon, \epsilon) + \frac{1 + 2\epsilon - 6\epsilon^2}{[12]^{1-\epsilon}[31]} J(1+\epsilon, 1-3\epsilon, 2) \\
&+ \left(\frac{-2 + 2\epsilon}{[31][23]^{2-\epsilon}} + \frac{-2 + 2\epsilon}{[31]^2[23]^{1-\epsilon}} + \frac{[12](2 - 2\epsilon)}{[31]^2[23]^{2-\epsilon}} \right) J(2, 1-2\epsilon, 0) \\
&+ \frac{2\epsilon - 4\epsilon^2}{[12]^{1-\epsilon}[31]^2} J(\epsilon, 3-3\epsilon, 0) \\
&+ \left(\frac{-2 + 6\epsilon^2}{[12]^{1-\epsilon}[31]} + \frac{(-\epsilon)[23]}{[12]^{1-\epsilon}[31]^2} + \frac{-2 + 5\epsilon - 2\epsilon^2}{[12]^{-\epsilon}[31]^2} \right) J(\epsilon, 2-3\epsilon, 2) \\
&+ \left(\frac{(-2\epsilon + 2\epsilon^2)[23]}{[12]^{1-\epsilon}[31]^2} + \frac{\epsilon - 2\epsilon^2}{[12]^{1-\epsilon}[31]} + \frac{4 - 11\epsilon + 2\epsilon^2}{[12]^{-\epsilon}[31]^2} \right) J(\epsilon, 3-3\epsilon, 1) \\
&+ \left(\frac{4 - 18\epsilon + 26\epsilon^2}{[31]^2[23]^{-\epsilon}} + \frac{-\epsilon + 4\epsilon^2}{[31][23]^{1-\epsilon}} + \frac{(3\epsilon - 10\epsilon^2)[21]}{[31]^2[23]^{1-\epsilon}} \right) J(1, 3-3\epsilon, \epsilon) \\
&+ \frac{(\epsilon)[23]}{[12]^{1-\epsilon}[31]^2} J(\epsilon - 1, 3-3\epsilon, 2) \\
&+ \left[\frac{(-2\epsilon + 6\epsilon^2)[21]}{[31]^2[23]^{1-\epsilon}} + \frac{(-\epsilon + 3\epsilon^2)[21]}{[31][23]^{2-\epsilon}} + \frac{[12]^2(\epsilon - 3\epsilon^2)}{[31]^2[23]^{2-\epsilon}} \right] J(2, 3-3\epsilon, \epsilon - 1) \\
&+ \frac{\epsilon - 3\epsilon^2}{[31]^2[23]^{1-\epsilon}} J(0, 2-3\epsilon, 1+\epsilon) + \frac{-\epsilon + 3\epsilon^2}{[31]^2[23]^{1-\epsilon}} J(0, 3-3\epsilon, \epsilon)
\end{aligned}
$$

$$+ \left(\frac{-1 + 3\epsilon - 2\epsilon^2}{[31][23]^{2-\epsilon}} + \frac{-1 + 3\epsilon - 2\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{(1 - 3\epsilon + 2\epsilon^2)[12]}{[31]^2[23]^{2-\epsilon}} \right) J(1, 2 - 2\epsilon, 0) \Bigg\}$$

$$= \frac{A(1, 1 - 2\epsilon, 1)A(1 + \epsilon, 1 - 3\epsilon, 1)}{8\epsilon^2(1 + 2\epsilon)(1 - 2\epsilon)^2} \Bigg\{ \frac{-2\epsilon + 10\epsilon^2}{[12]^{1-2\epsilon}[23]^{1-2\epsilon}[31]^{2+2\epsilon}}$$

$$+ \frac{3 + 7\epsilon + 3\epsilon^2}{[12]^{2-2\epsilon}[23]^{1-2\epsilon}[13]^{1+2\epsilon}} + \frac{-5 + 8\epsilon - 18\epsilon^2}{[12]^{2-2\epsilon}[23]^{-2\epsilon}[13]^{2+2\epsilon}} + \frac{-6\epsilon + \epsilon^2}{[31]^2[12]^{2-2\epsilon}}$$

$$+ \frac{2 - 4\epsilon - 5\epsilon^2}{[12]^{1-2\epsilon}[23]^{2-2\epsilon}[31]^{1+2\epsilon}} + \frac{2 - 4\epsilon}{[12]^{-2\epsilon}[23]^{2-2\epsilon}[31]^{2+2\epsilon}}$$

$$+ \frac{\epsilon - (1/2)\epsilon^2}{[12]^{2-2\epsilon}[23]^{2-2\epsilon}[31]^{2\epsilon}} + \frac{\epsilon + \epsilon^2/2}{[31]^2[23]^{2-2\epsilon}} \Bigg\}$$

$$+ \frac{A^2(1, 1 - 2\epsilon, 1)}{4\epsilon^2(1 + 2\epsilon)(1 - 2\epsilon)^2} \Bigg\{ \frac{1 - \epsilon}{[12]^{1-\epsilon}[23]^{2-\epsilon}[31]} + \frac{1 - \epsilon}{[12]^{1-\epsilon}[23]^{1-\epsilon}[31]^2}$$

$$- \frac{1 - \epsilon}{[12]^{-\epsilon}[23]^{2-\epsilon}[31]^2} \Bigg\} . \tag{B.1}$$

The terms in the sums $I_b^{(D+1)}$ and $I_b^{(D+2)}$ can be calculated also by using the uniqueness method, but only after having represented the J-terms as derivatives of integrals $J(\alpha_1, \alpha_2, \alpha_3)$ with $\sum \alpha_j = D$.

$$I_b^{(D+1)} + I_b^{(D+2)} = \frac{A(1, 1 - 2\epsilon, 1)}{4\epsilon(1 + 2\epsilon)(1 - 2\epsilon)^2} \Bigg\{ \frac{-11 + 13\epsilon + 32\epsilon^2}{[31][12]^{-\epsilon}} J(2 + \epsilon, 2 - 3\epsilon, 1)$$

$$+ \left[\frac{8 - 16\epsilon + 6\epsilon^2}{[31][23]^{-\epsilon}} + \frac{(2\epsilon - 5\epsilon^2)[21]}{[31][23]^{1-\epsilon}} \right] J(2, 2 - 3\epsilon, 1 + \epsilon)$$

$$+ \left(\frac{(2\epsilon - 2\epsilon^2)[23]}{[12]^{-\epsilon}[31]^2} + \frac{-4 + 13\epsilon - 6\epsilon^2}{[12]^{-\epsilon}[31]} + \frac{-4 + 13\epsilon - 6\epsilon^2}{[12]^{-\epsilon-1}[31]^2} \right) J(1 + \epsilon, 3 - 3\epsilon, 1)$$

$$+ \frac{(\epsilon)[23]}{[12]^{-\epsilon}[31]} J(1 + \epsilon, 3 - 3\epsilon, 2) - \left(\frac{(\epsilon)[23]}{[12]^{1-\epsilon}[31]} + \frac{(\epsilon)[23]}{[12]^{-\epsilon}[31]^2} \right) J(\epsilon, 3 - 3\epsilon, 2)$$

$$+ \frac{-1 - \epsilon + 4\epsilon^2}{[12]^{1-\epsilon}} J(2 + \epsilon, 1 - 3\epsilon, 2) + \frac{-1 - \epsilon + 4\epsilon^2}{[12]^{-\epsilon}} J(2 + \epsilon, 2 - 3\epsilon, 2)$$

$$+ \left(\frac{1 + \epsilon - 4\epsilon^2}{[12]^{1-\epsilon}} + \frac{3 - 4\epsilon - 2\epsilon^2}{[12]^{-\epsilon}[31]} + \frac{(\epsilon)[23]}{[12]^{1-\epsilon}[31]} \right) J(1 + \epsilon, 2 - 3\epsilon, 2)$$

$$+ \frac{4 - 14\epsilon + 8\epsilon^2}{[31][12]^{1-\epsilon}} J(2 + \epsilon, 3 - 3\epsilon, 1)$$

$$+ \frac{(-4 + 16\epsilon - 19\epsilon^2)[21]}{[31][23]^{-\epsilon}} J(2, 3 - 3\epsilon, 1 + \epsilon)$$

$$+ \left(\frac{(4 - 16\epsilon + 19\epsilon^2)[21]}{[31]^2[23]^{-\epsilon}} + \frac{4 - 16\epsilon + 19\epsilon^2}{[31][23]^{-\epsilon}} \right) J(1, 3 - 3\epsilon, 1 + \epsilon)$$

$$+ \left[\frac{(-4 + 18\epsilon - 26\epsilon^2)[21]}{[31]^2[23]^{-\epsilon}} + \frac{(\epsilon - 4\epsilon^2)[21]}{[31][23]^{1-\epsilon}} + \frac{[12]^2(-2\epsilon + 7\epsilon^2)}{[31]^2[23]^{1-\epsilon}} \right] J(2, 3 - 3\epsilon, \epsilon) \Bigg\}$$

$$= \frac{A(1, 1 - 2\epsilon, 1)A(1 - 3\epsilon, 1, 1 + \epsilon)}{8\epsilon^2(1 + 2\epsilon)(1 - 2\epsilon)^2} \Bigg\{ \frac{2 - (17/2)\epsilon - (35/4)\epsilon^2}{[12]^{1-2\epsilon}[23]^{1-2\epsilon}[13]^{2+2\epsilon}}$$

$$+ \frac{-3 - (3/2)\epsilon - (7/4)\epsilon^2}{[12]^{2-2\epsilon}[23]^{1-2\epsilon}[13]^{1+2\epsilon}} + \frac{5 - 5\epsilon + (39/2)\epsilon^2}{[12]^{2-2\epsilon}[23]^{-2\epsilon}[13]^{2+2\epsilon}}$$

$$+ \frac{1 - 4\epsilon + (21/2)\epsilon^2}{[12]^{1-2\epsilon}[23]^{2-2\epsilon}[31]^{1+2\epsilon}} + \frac{-4 + (15/2)\epsilon - (19/4)\epsilon^2}{[12]^{-2\epsilon}[23]^{2-2\epsilon}[31]^{2+2\epsilon}} + \frac{(5/2)\epsilon - (7/4)\epsilon^2}{[12]^{2-2\epsilon}[23]^{2-2\epsilon}[31]^{2\epsilon}} \Bigg\}. \text{(B.2)}$$

C. Useful relations between various scalar single integrals

In this appendix we provide formulas used in section 4. The following formula results from applying the corresponding IBP procedure:

$$
\begin{aligned}
J(1, 3 - 3\epsilon, \epsilon - 1) &= -\frac{1}{2 - 3\epsilon} \left[J(2, 2 - 3\epsilon, \epsilon - 1) - [12]J(2, 3 - 3\epsilon, \epsilon - 1) \right.\\
&\quad \left. + (\epsilon - 1)\left(J(1, 2 - 3\epsilon, \epsilon) - [23]J(1, 3 - 3\epsilon, \epsilon) \right) \right] = \\
&\quad -\frac{1}{2 - 3\epsilon} \left[J(2, 2 - 3\epsilon, \epsilon - 1) - \frac{A(2, 3 - 3\epsilon, \epsilon - 1)}{[12]^{2-2\epsilon}[23]^{-\epsilon}[31]^{-1+2\epsilon}} \right.\\
&\quad \left. + (\epsilon - 1)\left(J(1, 2 - 3\epsilon, \epsilon) - \frac{A(1, 3 - 3\epsilon, \epsilon)}{[12]^{2-2\epsilon}[23]^{-\epsilon}[31]^{-1+2\epsilon}} \right) \right]. \text{(C.1)}
\end{aligned}
$$

Other useful formulas can be taken from ref. [9] (appendix A there, obtained by IBP procedure):

$$
\begin{aligned}
J(1 - 3\epsilon, 2, \epsilon) &= -(1 - 3\epsilon)J(2 - 3\epsilon, 1, \epsilon) - \epsilon J(1 - 3\epsilon, 1, 1 + \epsilon) \\
&\quad - \frac{A(1, 1 - 3\epsilon, 1 + \epsilon)}{2\epsilon} \frac{1}{[12]^{1-2\epsilon}[13]^{-\epsilon}[23]^{2\epsilon}}, \quad \text{(C.2)}
\end{aligned}
$$

$$
\begin{aligned}
J(2, 1 - 3\epsilon, \epsilon) &= -(1 - 3\epsilon)J(1, 2 - 3\epsilon, \epsilon) - \epsilon J(1, 1 - 3\epsilon, 1 + \epsilon) \\
&\quad - \frac{A(1, 1 - 3\epsilon, 1 + \epsilon)}{2\epsilon} \frac{1}{[12]^{1-2\epsilon}[23]^{-\epsilon}[31]^{2\epsilon}}, \quad \text{(C.3)}
\end{aligned}
$$

$$
\begin{aligned}
J(\epsilon, 1 - 3\epsilon, 2) &= -(1 - 3\epsilon)J(\epsilon, 2 - 3\epsilon, 1) - \epsilon J(1 + \epsilon, 1 - 3\epsilon, 1) \\
&\quad - \frac{A(1, 1 - 3\epsilon, 1 + \epsilon)}{2\epsilon} \frac{1}{[23]^{1-2\epsilon}[12]^{-\epsilon}[31]^{2\epsilon}}. \quad \text{(C.4)}
\end{aligned}
$$

The following useful relation can be directly derived:

$$
\begin{aligned}
J(2, 2 - 3\epsilon, \epsilon - 1) &\equiv \int Dx \frac{1}{[x1]^2[x2]^{2-3\epsilon}[x3]^{\epsilon-1}} = \int Dx \frac{[x3]}{[x1]^2[x2]^{2-3\epsilon}[x3]^\epsilon} \\
&= \int Dx \frac{[x1] + [13] + 2(x1)(13)}{[x1]^2[x2]^{2-3\epsilon}[x3]^\epsilon} \\
&= J(1, 2 - 3\epsilon, \epsilon) + [31]J(2, 2 - 3\epsilon, \epsilon) + 2(13)\int Dx \frac{(x1)}{[x1]^2[x2]^{2-3\epsilon}[x3]^\epsilon} \\
&= J(1, 2 - 3\epsilon, \epsilon) + [31]J(2, 2 - 3\epsilon, \epsilon) + (13)_\mu \partial_\mu^{(1)} J(1, 2 - 3\epsilon, \epsilon) \\
&= J(1, 2 - 3\epsilon, \epsilon) + \frac{A(2, 2 - 3\epsilon, \epsilon)}{[12]^{2-2\epsilon}[23]^{-\epsilon}[31]^{-1+2\epsilon}} + (13)_\mu \partial_\mu^{(1)} J(1, 2 - 3\epsilon, \epsilon) \\
&= J(1, 2 - 3\epsilon, \epsilon) - \frac{(1 - 2\epsilon)}{2\epsilon(1 - 3\epsilon)} \frac{A(1, 1 - 3\epsilon, 1 + \epsilon)}{[12]^{2-2\epsilon}[23]^{-\epsilon}[31]^{-1+2\epsilon}} + (13)_\mu \partial_\mu^{(1)} J(1, 2 - 3\epsilon, \epsilon),
\end{aligned}
$$

$$
\begin{aligned}
J(\epsilon - 1, 2 - 3\epsilon, 2) &= \\
&= J(\epsilon, 2 - 3\epsilon, 1) - \frac{(1 - 2\epsilon)}{2\epsilon(1 - 3\epsilon)} \frac{A(1, 1 - 3\epsilon, 1 + \epsilon)}{[23]^{2-2\epsilon}[12]^{-\epsilon}[31]^{-1+2\epsilon}} + (31)_\mu \partial_\mu^{(3)} J(\epsilon, 2 - 3\epsilon, 1). \text{(C.5)}
\end{aligned}
$$

For $J(2 - 3\epsilon, 1, \epsilon)$ we use specific formulas of ref. [9] which were obtained by applying the GPT:

$$J(2 - 3\epsilon, 1, \epsilon) = \frac{1}{[12]^{1-\epsilon}} A(1 - 3\epsilon, 1, 1 + \epsilon) \frac{1}{2(1 - 3\epsilon)} \times$$

$$\times \left[\frac{1}{\epsilon} - \ln \frac{[13]}{[12]} - \epsilon \left(\frac{1}{2} \ln \frac{[12][13]}{[23]^2} \ln \frac{[13]}{[12]} + ([12] + [23] - [13]) J(1, 1, 1) \right) \right],$$

$$J(1, 2 - 3\epsilon, \epsilon) = \frac{1}{[12]^{1-\epsilon}} A(1 - 3\epsilon, 1, 1 + \epsilon) \frac{1}{2(1 - 3\epsilon)} \times$$

$$\times \left[\frac{1}{\epsilon} - \ln \frac{[23]}{[12]} - \epsilon \left(\frac{1}{2} \ln \frac{[12][23]}{[13]^2} \ln \frac{[23]}{[12]} + ([12] + [13] - [23]) J(1, 1, 1) \right) \right],$$

$$J(\epsilon, 2 - 3\epsilon, 1) = \frac{1}{[23]^{1-\epsilon}} A(1 - 3\epsilon, 1, 1 + \epsilon) \frac{1}{2(1 - 3\epsilon)} \times$$

$$\times \left[\frac{1}{\epsilon} - \ln \frac{[12]}{[23]} - \epsilon \left(\frac{1}{2} \ln \frac{[23][12]}{[13]^2} \ln \frac{[12]}{[23]} + ([23] + [13] - [12]) J(1, 1, 1) \right) \right]. \quad \text{(C.6)}$$

D. Calculation of $I_b^{(\partial J,1)}$ and $I_b^{(\partial J,2)}$ terms

In this appendix we apply formulas (C.6) and obtain for the term in eq. (5.1) proportional to $\partial_\mu^{(1)} J(1, 2 - 3\epsilon, \epsilon)$

$$I_b^{(\partial J,1)} \equiv \frac{A(1, 1 - 2\epsilon, 1)}{4\epsilon(1 + 2\epsilon)(1 - 2\epsilon)^2} \left[\frac{3\epsilon - (5/2)\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{(3/2)\epsilon - (5/4)\epsilon^2}{[31][23]^{2-\epsilon}} \right.$$

$$\left. + \frac{[12](-(3/2)\epsilon + (5/4)\epsilon^2)}{[31]^2[23]^{2-\epsilon}} \right] (13)_\mu \partial_\mu^{(1)} J(1, 2 - 3\epsilon, \epsilon) =$$

$$- \frac{A(1, 1 - 2\epsilon, 1) A(1 - 3\epsilon, 1, 1 + \epsilon)}{8\epsilon(1 + 2\epsilon)(1 - 2\epsilon)^2} \left[\frac{9/2}{[12]^{1-\epsilon}[31]^2[23]^{1-\epsilon}} + \frac{3/2}{[12]^{2-\epsilon}[31][23]^{1-\epsilon}} \right.$$

$$\left. + \frac{-3}{[12]^{2-\epsilon}[31]^2[23]^{-\epsilon}} + \frac{3/2}{[12]^{2-\epsilon}[23]^{2-\epsilon}} + \frac{-3/2}{[12]^{-\epsilon}[31]^2[23]^{2-\epsilon}} \right]$$

$$- \frac{1}{8} \left[\frac{21/4}{[12][23][31]^2} + \frac{7/4}{[12]^2[23][31]} + \frac{-7/2}{[12]^2[31]^2} + \frac{7/4}{[12]^2[23]^2} + \frac{-7/4}{[31]^2[23]^2} \right]$$

$$- \frac{3}{16} \left[\frac{2}{[31]^2[23]} + \frac{1}{[31][23]^2} - \frac{[12]}{[31]^2[23]^2} \right] (13)_\mu \partial_\mu^{(1)} \frac{1}{[12]} \ln \frac{[23]}{[12]}, \quad \text{(D.1)}$$

and for the term in eq. (5.1) proportional to $\partial_\mu^{(3)} J(\epsilon, 2 - 3\epsilon, 1)$

$$I_b^{(\partial J,2)} \equiv \frac{A(1, 1 - 2\epsilon, 1)}{4\epsilon(1 + 2\epsilon)(1 - 2\epsilon)^2} \frac{\left(1 - (5/2)\epsilon^2\right)}{[12]^{1-\epsilon}[31]^2} (31)_\mu \partial_\mu^{(3)} J(\epsilon, 2 - 3\epsilon, 1)$$

$$= \frac{A(1, 1 - 2\epsilon, 1)}{4\epsilon(1 + 2\epsilon)(1 - 2\epsilon)^2} \frac{\left(1 - (5/2)\epsilon^2\right)}{[12]^{1-\epsilon}[31]^2} (31)_\mu \partial_\mu^{(3)} \left\{ \frac{1}{[23]^{1-\epsilon}} A(1 - 3\epsilon, 1, 1 + \epsilon) \frac{1}{2(1 - 3\epsilon)} \times \right.$$

$$\left. \times \left[\frac{1}{\epsilon} - \ln \frac{[12]}{[23]} - \epsilon \left(\frac{1}{2} \ln \frac{[23][12]}{[13]^2} \ln \frac{[12]}{[23]} + ([23] + [13] - [12]) J(1, 1, 1) \right) \right] \right\}$$

$$= - \frac{A(1, 1 - 2\epsilon, 1) A(1 - 3\epsilon, 1, 1 + \epsilon)}{8\epsilon^2(1 + 2\epsilon)(1 - 2\epsilon)^2} \left[\frac{1}{[12]^{1-\epsilon}[23]^{2-\epsilon}[31]} \right.$$

$$
+ \frac{1}{[12]^{1-\epsilon}[23]^{1-\epsilon}[31]^2} - \frac{1}{[12]^{-\epsilon}[23]^{2-\epsilon}[31]^2} \Bigg]
$$

$$
+ \frac{A(1,1-2\epsilon,1)A(1-3\epsilon,1,1+\epsilon)}{8\epsilon(1+2\epsilon)(1-2\epsilon)^2} \Bigg[\frac{1}{[12]^{1-\epsilon}[23]^{2-\epsilon}[31]}
$$

$$
+ \frac{1}{[12]^{1-\epsilon}[23]^{1-\epsilon}[31]^2} - \frac{1}{[12]^{-\epsilon}[23]^{2-\epsilon}[31]^2} \Bigg] \ln \frac{[12]}{[23]}
$$

$$
- \frac{A(1,1-2\epsilon,1)A(1-3\epsilon,1,1+\epsilon)}{8\epsilon(1+2\epsilon)(1-2\epsilon)^2} \Bigg[\frac{1}{[12]^{1-\epsilon}[23]^{2-\epsilon}[31]}
$$

$$
+ \frac{1}{[12]^{1-\epsilon}[23]^{1-\epsilon}[31]^2} - \frac{1}{[12]^{-\epsilon}[23]^{2-\epsilon}[31]^2} \Bigg]
$$

$$
- \frac{1}{8} \frac{1}{[12][31]^2} (31)_\mu \partial_\mu^{(3)} \frac{1}{[23]} \left(\frac{1}{2} \ln \frac{[23][12]}{[13]^2} \ln \frac{[12]}{[23]} + ([23]+[13]-[12])J(1,1,1) \right)
$$

$$
- \frac{1}{16} \left[\frac{1}{[12][23]^2[31]} + \frac{1}{[12][23][31]^2} - \frac{1}{[23]^2[31]^2} \right]
$$

$$
+ \frac{1}{4} \left[\frac{1}{[12][23]^2[31]} + \frac{1}{[12][23][31]^2} - \frac{1}{[23]^2[31]^2} \right] \ln \frac{[12]}{[23]} \ . \tag{D.2}
$$

E. Cancellation of poles in ϵ in I_b

Collecting (5.2), (D.1) and (D.2) in eq. (5.1), we obtain the final expression for the diagram (b) in D dimensions

$$
I_b = \frac{A(1,1-2\epsilon,1)}{4\epsilon(1+2\epsilon)(1-2\epsilon)^2} \Bigg\{ \frac{5\epsilon-12\epsilon^2}{[31]^2[12]^{1-\epsilon}} J(\epsilon,2-3\epsilon,1)
$$

$$
+ \left[\frac{-1+9\epsilon-17\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{-1+8\epsilon-(23/2)\epsilon^2}{[31][23]^{2-\epsilon}} + \frac{(1-8\epsilon+(23/2)\epsilon^2)[21]}{[31]^2[23]^{2-\epsilon}} \right] J(1,2-3\epsilon,\epsilon)
$$

$$
+ \left[\frac{3\epsilon-(5/2)\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{-(3/2)\epsilon+(5/4)\epsilon^2}{[31][23]^{2-\epsilon}} + \frac{[12](-(3/2)\epsilon+(5/4)\epsilon^2)}{[31]^2[23]^{2-\epsilon}} \right] \times
$$

$$
\times (13)_\mu \partial_\mu^{(1)} J(1,2-3\epsilon,\epsilon) + \frac{1-(5/2)\epsilon^2}{[12]^{1-\epsilon}[31]^2} (31)_\mu \partial_\mu^{(3)} J(\epsilon,2-3\epsilon,1) \Bigg\}
$$

$$
- \frac{1}{2} \left[\frac{1}{[31]^2[23]} + \frac{1}{[31][23]^2} - \frac{[12]}{[31]^2[23]^2} \right] J(1,1,1)
$$

$$
+ \frac{1}{8} \left\{ \frac{-27/2}{[12][23][31]^2} + \frac{5}{[12][23]^2[31]} + \frac{1/4}{[12]^2[23][31]} + \frac{-11/4}{[12]^2[23]^2} + \frac{-9/4}{[23]^2[31]^2} + \frac{5/2}{[12]^2[31]^2} \right\}
$$

$$
+ \frac{A(1,1-2\epsilon,1)A(1,1-3\epsilon,1+\epsilon)}{8\epsilon(1+2\epsilon)(1-2\epsilon)^2} \Bigg\{ \frac{-7/2}{[12]^{1-2\epsilon}[23]^{1-2\epsilon}[31]^{2+2\epsilon}} + \frac{-2}{[12]^{1-2\epsilon}[23]^{2-2\epsilon}[31]^{1+2\epsilon}}
$$

$$
+ \frac{-1/2}{[12]^{-2\epsilon}[23]^{2-2\epsilon}[31]^{2+2\epsilon}} + \frac{3/2}{[12]^{2-2\epsilon}[23]^{1-2\epsilon}[31]^{1+2\epsilon}} + \frac{3/2}{[12]^{2-2\epsilon}[23]^{2-2\epsilon}[31]^{2\epsilon}}
$$

$$
+ \frac{3}{[12]^{2-2\epsilon}[23]^{-2\epsilon}[31]^{2+2\epsilon}} + \frac{-6}{[31]^2[12]^{2-2\epsilon}} + \frac{1}{[31]^2[23]^{2-2\epsilon}} \Bigg\}
$$

$$
+ \frac{A^2(1,1-2\epsilon,1)}{4\epsilon^2(1+2\epsilon)(1-2\epsilon)^2} \Bigg\{ \frac{1-\epsilon}{[12]^{1-\epsilon}[23]^{2-\epsilon}[31]} + \frac{1-\epsilon}{[12]^{1-\epsilon}[23]^{1-\epsilon}[31]^2} - \frac{1-\epsilon}{[12]^{-\epsilon}[23]^{2-\epsilon}[31]^2} \Bigg\} \tag{E.1}
$$

Taking the limit $\epsilon \to 0$, the poles in ϵ disappear and the result is

$$
\begin{aligned}
I_b(\epsilon \to 0) = {} & \frac{1}{8} \left[\frac{7/2}{[12][23][31]^2} \ln \frac{[31]^2}{[12][23]} + \frac{2}{[12][23]^2[31]} \ln \frac{[31]^2}{[12][23]} \right. \\
& \quad + \frac{-3/2}{[12]^2[23][31]} \ln \frac{[31]^2}{[12][23]} + \frac{-1/2}{[23]^2[31]^2} \ln \frac{[12]^3}{[23][31]^2} \\
& \quad \left. + \frac{3}{[12]^2[31]^2} \ln \frac{[23]^3}{[12][31]^2} + \frac{-3/2}{[12]^2[23]^2} \ln \frac{[31]^2}{[12][23]} \right] \\
& + \frac{1}{16} \left[\frac{1}{[12][31]^2[23]} + \frac{1}{[12][31][23]^2} + \frac{-1}{[31]^2[23]^2} \right] \ln \frac{[12][23]}{[13]^2} \ln \frac{[23]}{[12]} \\
& + \frac{1}{8} \left[\frac{-2}{[31]^2[23]} + \frac{3[12]}{[31]^2[23]^2} + \frac{1}{[12][23]^2} - \frac{1}{[31]^2[12]} + \frac{-4}{[31][23]^2} \right] J(1,1,1) \\
& - \frac{3}{16} \left[\frac{2}{[31]^2[23]} + \frac{1}{[31][23]^2} - \frac{[12]}{[31]^2[23]^2} \right] (13)_\mu \partial_\mu^{(1)} \frac{1}{[12]} \ln \frac{[23]}{[12]} \\
& - \frac{1}{8} \frac{1}{[12][31]^2} (31)_\mu \partial_\mu^{(3)} \frac{1}{[23]} \left(\frac{1}{2} \ln \frac{[23][12]}{[13]^2} \ln \frac{[12]}{[23]} + ([23]+[13]-[12]) J(1,1,1) \right) \\
& + \frac{1}{8} \left[\frac{7}{[12][23]^2[31]} + \frac{3}{[12][23][31]^2} + \frac{-7}{[23]^2[31]^2} \right] \ln \frac{[12]}{[23]} \\
& + \frac{1}{16} \left\{ \frac{-13}{[12][23][31]^2} + \frac{16}{[12][23]^2[31]} + \frac{-3}{[12]^2[23][31]} + \frac{-9}{[12]^2[23]^2} \right. \\
& \qquad \left. + \frac{-7}{[23]^2[31]^2} + \frac{12}{[12]^2[31]^2} \right\} .
\end{aligned}
\tag{E.2}
$$

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